Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Gaussian Elimination
Simultaneous Linear Equations
COMPLETE SOLUTION SET

1. The goal of forward elimination steps in the Naïve Gauss elimination method is to reduce the coefficient matrix to a (an) ______________ matrix.

(A) diagonal
(B) identity
(C) lower triangular
(D) upper triangular

Solution
The correct answer is (D).

By reducing the coefficient matrix to an upper triangular matrix, starting from the last equation, each equation can be reduced to one equation-one unknown to be solved by back substitution.

2. Division by zero during forward elimination steps in Naïve Gaussian elimination of the set of equations $[A][X]=[C]$ implies the coefficient matrix $[A]$

(A) is invertible
(B) is nonsingular
(C) may be singular or nonsingular
(D) is singular

Solution
The correct answer is (C).

Division by zero during forward elimination does not relate to whether or not the coefficient matrix is singular or nonsingular. For example

$$\begin{bmatrix} 0 & 2 & 3 \\ 2 & 3 & 7 \\ 1 & 5 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 5 \\ 12 \\ 12 \end{bmatrix}$$

would give a division by zero error in the first step of forward elimination. However, the coefficient matrix in this case is nonsingular.

In another example

$$\begin{bmatrix} 0 & 2 & 3 \\ 2 & 3 & 7 \\ 4 & 6 & 14 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 5 \\ 12 \\ 12 \end{bmatrix}$$

would also give a division by zero error in the first step of forward elimination. In this case the coefficient matrix is singular.

3. Using a computer with four significant digits with chopping, the Naïve Gauss elimination solution to

$$0.0030x_1 + 55.23x_2 = 58.12$$
$$6.239x_1 - 7.123x_2 = 47.23$$

is

(A) $x_1 = 26.66;\ x_2 = 1.051$
(B) $x_1 = 8.769;\ x_2 = 1.051$
(C) $x_1 = 8.800;\ x_2 = 1.000$
(D) $x_1 = 8.771;\ x_2 = 1.052$

Solution

The correct answer is (A).

$$\begin{bmatrix} 0.0030 & 55.23 \\ 6.239 & -7.123 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 58.12 \\ 47.23 \end{bmatrix}$$

Forward Elimination: Divide *Row 1* by 0.0030 and then multiply it by 6.239

$$\left[\frac{Row1}{0.0030}\right]\times(6.239) = \text{[Row 1]} \times 2079 \text{ gives } \textit{Row 1} \text{ as}$$

$$\begin{bmatrix} 6.239 & 1.148\times10^5 \end{bmatrix} \qquad \begin{bmatrix} 1.208\times10^5 \end{bmatrix}$$

Subtract the result from *Row 2*.

$$\begin{bmatrix} 0.0030 & 55.23 \\ 0 & -1.148\times10^5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 58.12 \\ -1.207\times10^5 \end{bmatrix}$$

Back substitution: From the second equation

$$\left(-1.148\times10^5\right)\times x_2 = -1.207\times10^5$$

$$x_2 = \frac{-1.207\times10^5}{-1.148\times10^5}$$

$$= 1.051$$

Substituting the value of x_2 in the first equation

$$0.0030x_1 + 55.23x_2 = 58.12$$

$$x_1 = \frac{58.12 - 55.23x_2}{0.0030}$$

$$= \frac{58.12 - 55.23(1.051)}{0.0030}$$

$$= \frac{58.12 - 58.04}{0.0030}$$

$$= \frac{0.08}{0.0030}$$

$$= 26.66$$

4. Using a computer with four significant digits with chopping, the Gaussian elimination with partial pivoting solution to

$$0.0030x_1 + 55.23x_2 = 58.12$$

$$6.239x_1 - 7.123x_2 = 47.23$$

is

(A) $x_1 = 26.66;\ x_2 = 1.051$
(B) $x_1 = 8.769;\ x_2 = 1.051$
(C) $x_1 = 8.800;\ x_2 = 1.000$
(D) $x_1 = 8.771;\ x_2 = 1.052$

Solution

The correct answer is (B).

$$\begin{bmatrix} 0.0030 & 55.23 \\ 6.239 & -7.123 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 58.12 \\ 47.23 \end{bmatrix}$$

Forward elimination:

Now for the first step of forward elimination, the absolute value of first column elements is

$$|0.0030|, |6.239|$$

or

$$0.0030,\ 6.239$$

So the largest absolute value is in *Row 2*. So as per Gaussian elimination with partial pivoting, the switch is between *Row 1* and *Row 2* to give

$$\begin{bmatrix} 6.239 & -7.123 \\ 0.0030 & 55.23 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 47.23 \\ 58.12 \end{bmatrix}$$

$\left[\frac{Row1}{6.239}\right] \times (0.0030) = [Row\ 1] \times 4.808 \times 10^{-4}$ gives *Row 1* as

$$\begin{bmatrix} 2.999\times10^{-3} & -3.424\times10^{-3} \end{bmatrix} \quad \begin{bmatrix} 2.270\times10^{-2} \end{bmatrix}$$

Subtract the result from *Row 2*.

$$\begin{bmatrix} 6.239 & -7.123 \\ 0 & 55.23 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 47.23 \\ 58.09 \end{bmatrix}$$

Note: This entry is zero because the algorithm for forward elimination treats it as zero.

Back substitution: From the second equation

$$55.23x_2 = 58.09$$

$$x_2 = \frac{58.09}{55.23}$$

$$= 1.051$$

Substituting the value of x_2 in the first equation

$$6.239x_1 - 7.123x_2 = 47.23$$

$$x_1 = \frac{47.23 + 7.123x_2}{6.239}$$

$$= \frac{47.23 + 7.123(1.051)}{6.239}$$

$$= \frac{47.23 + 7.486}{6.239}$$

$$= \frac{54.71}{6.239}$$

$$= 8.769$$

5. At the end of the forward elimination steps of the Naïve Gauss elimination method on the following equations

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 4.2857\times10^{7} & -5.4619\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ -6.5 & -0.15384 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857\times10^{7} & -3.6057\times10^{5} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -7.887\times10^{3} \\ 0 \\ 0.007 \\ 0 \end{bmatrix}$$

the resulting equations in matrix form are given by

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 0 & 3.7688\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ 0 & 0 & -26.9140 & 0.579684 \\ 0 & 0 & 0 & 5.62500\times10^{5} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -7.887\times10^{3} \\ 7.887\times10^{3} \\ 1.19530\times10^{-2} \\ 1.90336\times10^{4} \end{bmatrix}$$

The determinant of the original coefficient matrix is

(A) 0.00

(B) 4.2857×10^{7}

(C) 5.486×10^{19}

(D) -2.445×10^{20}

Solution

The correct answer is (D).

If a matrix is upper triangular, lower triangular or diagonal, then the determinant is

$$a_{11}\times a_{22}\times ....\times a_{nn} = \prod_{i=1}^{n} a_{ii}$$

Thus, the determinant, D, of the matrix is

$$D = \left(4.2857\times10^{7}\right)\times\left(3.7688\times10^{5}\right)\times\left(-26.9140\right)\times\left(5.62500\times10^{5}\right)$$

$$= -2.445\times10^{20}$$

6. The following data is given for the velocity of the rocket as a function of time. To find the velocity at $t = 21\,\text{s}$, you are asked to use a quadratic polynomial, $v(t) = at^2 + bt + c$ to approximate the velocity profile.

t	(s)	0	14	15	20	30	35
$v(t)$	(m/s)	0	227.04	362.78	517.35	602.97	901.67

The correct set of equations that will find a, b and c are

(A) $$\begin{bmatrix} 176 & 14 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 227.04 \\ 362.78 \\ 517.35 \end{bmatrix}$$

(B) $$\begin{bmatrix} 225 & 15 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 362.78 \\ 517.35 \\ 602.97 \end{bmatrix}$$

(C) $$\begin{bmatrix} 0 & 0 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 362.78 \\ 517.35 \end{bmatrix}$$

(D) $$\begin{bmatrix} 400 & 20 & 1 \\ 900 & 30 & 1 \\ 1225 & 35 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 517.35 \\ 602.97 \\ 901.67 \end{bmatrix}$$

Solution

The correct answer is (B).

First choose the three points closest to $t = 21\,\text{s}$ that also bracket it.

$$t_0 = 15\,\text{s},\ v(t_0) = 362.78\ \text{m/s}$$
$$t_1 = 20\,\text{s},\ v(t_1) = 517.35\ \text{m/s}$$
$$t_2 = 30\,\text{s},\ v(t_2) = 602.97\ \text{m/s}$$

Such that

$$v(15) = 362.78 = a(15)^2 + b(15) + c$$
$$v(20) = 517.35 = a(20)^2 + b(20) + c$$
$$v(30) = 602.97 = a(30)^2 + b(30) + c$$

This expands to

$$225a + 15b + c = 362.78$$
$$400a + 20b + c = 517.35$$
$$900a + 30b + c = 602.97$$
$$\begin{bmatrix} 225a + 15b + c \\ 400a + 20b + c \\ 900a + 30b + c \end{bmatrix} = \begin{bmatrix} 362.78 \\ 517.35 \\ 602.97 \end{bmatrix}$$

This can be rewritten as

$$\begin{bmatrix} 225 & 15 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 362.78 \\ 517.35 \\ 602.97 \end{bmatrix}$$