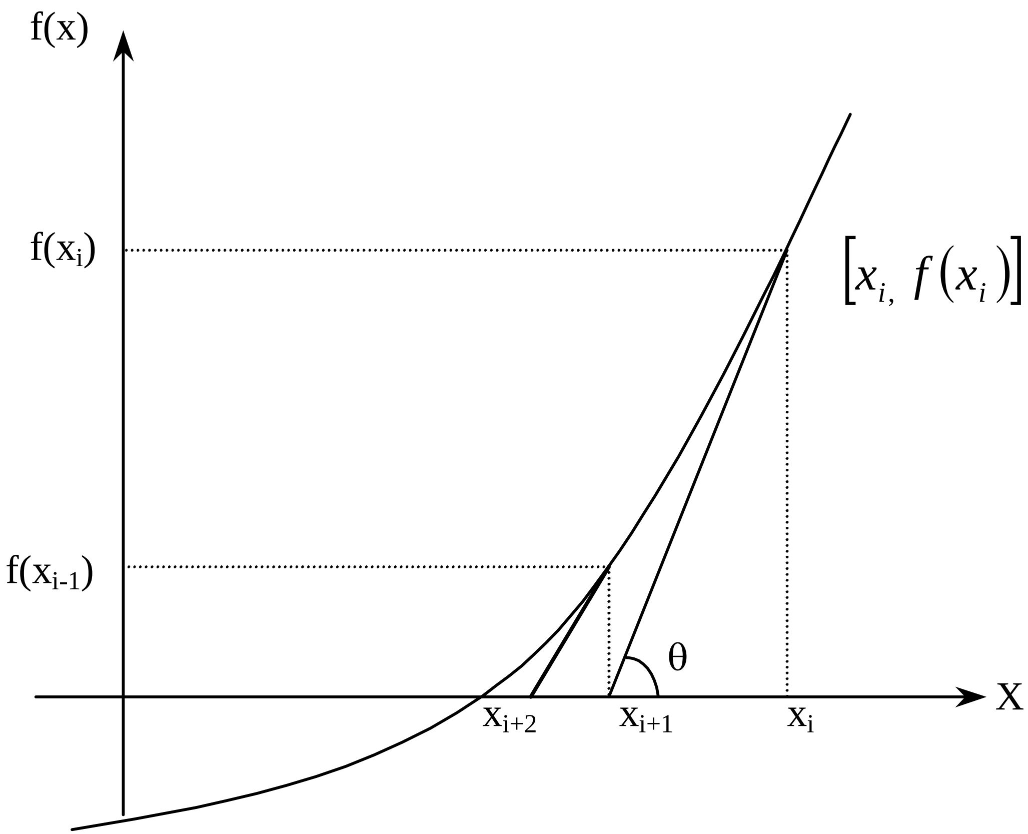
Roots of a Nonlinear Equation

Topic: Newton-Raphson Method

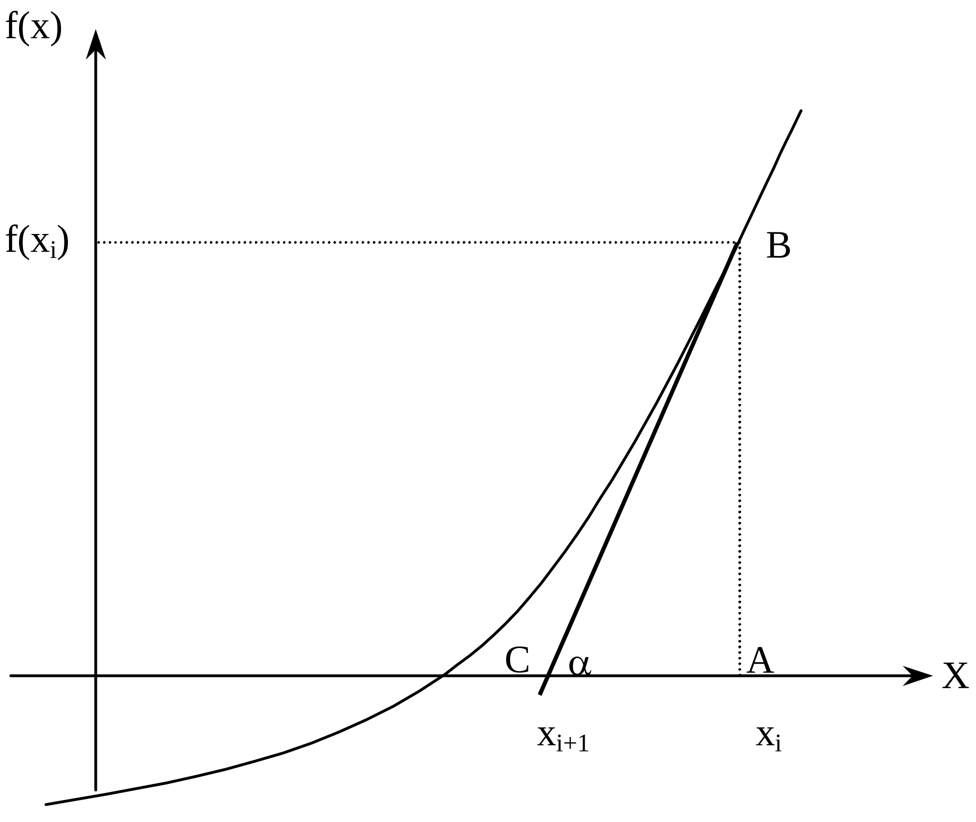
Major: General Engineering

Newton-Raphson Method



$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Derivation



$$\tan(\alpha) = \frac{AB}{AC}$$

$$f'(x_i) = \frac{f(x_i)}{x_i - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Algorithm for Newton-Raphson Method

Step 1

Evaluate $f'(x)$ symbolically

Step 2

Calculate the next estimate of the root

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Find the absolute relative approximate error

$$|\in_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$$

Step 3

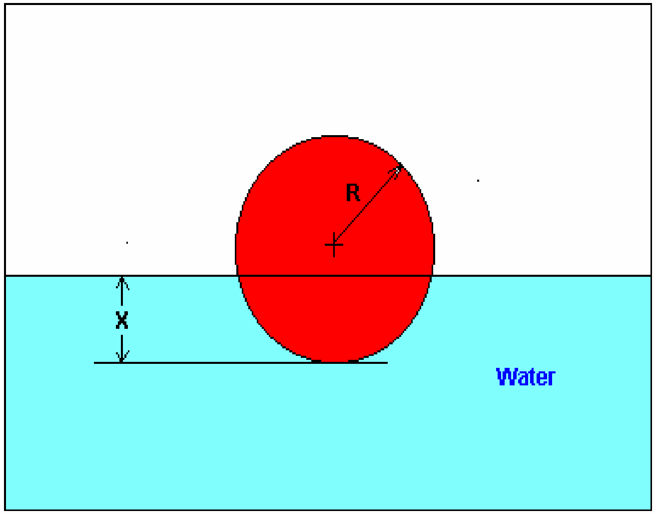
- Find if the absolute relative approximate error is greater than the pre-specified relative error tolerance.

- If so, go back to step 2, else stop the algorithm.

- Also check if the number of iterations has exceeded the maximum number of iterations.

Example

- You are working for 'DOWN THE TOILET COMPANY' that makes floats for ABC commodes. The ball has a specific gravity of 0.6 and has a radius of 5.5 cm. You are asked to find the distance to which the ball will get submerged when floating in water.



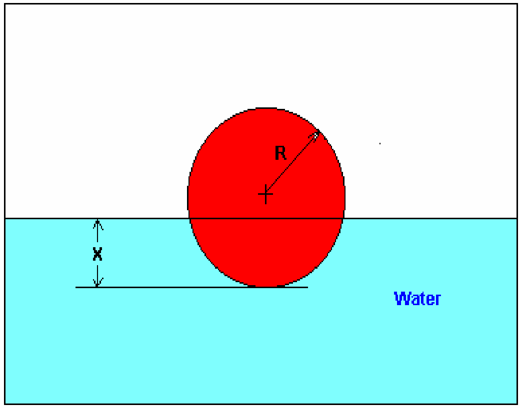
Solution

The equation that gives the depth 'x' to which the ball is submerged under water is given by

$$f(x) = x^3 - 0.165x^2 + 3.993\text{x}10^{-4}$$

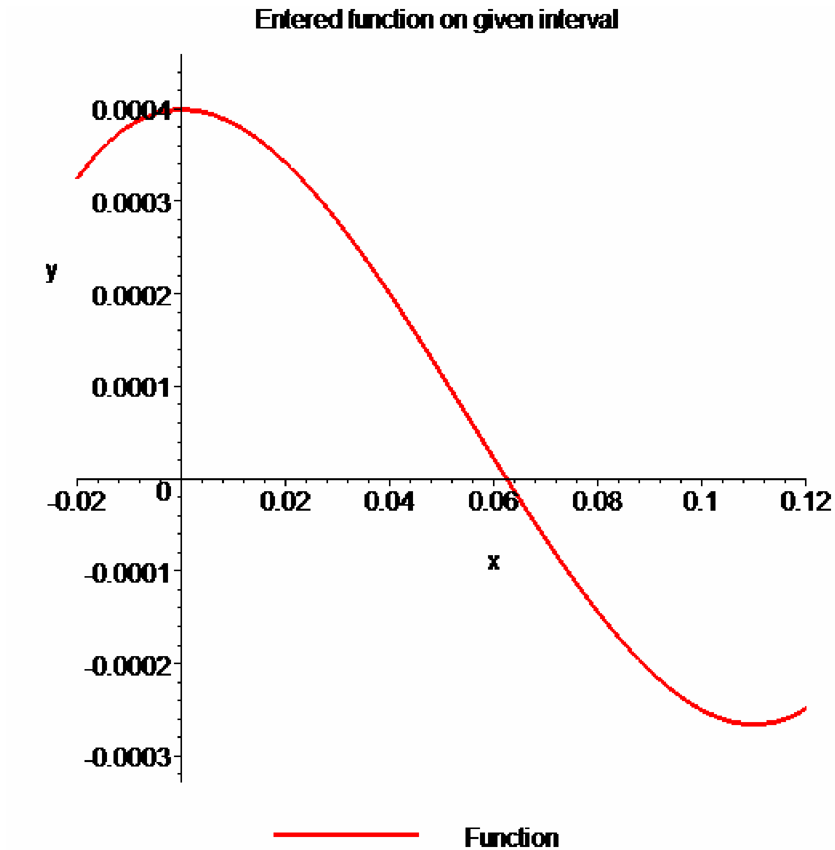
$$f'(x) = 3x^2 - 0.33x$$

Use the Newton's method of finding roots of equations to find the depth 'x' to which the ball is submerged under water. Conduct three iterations to estimate the root of the above equation.

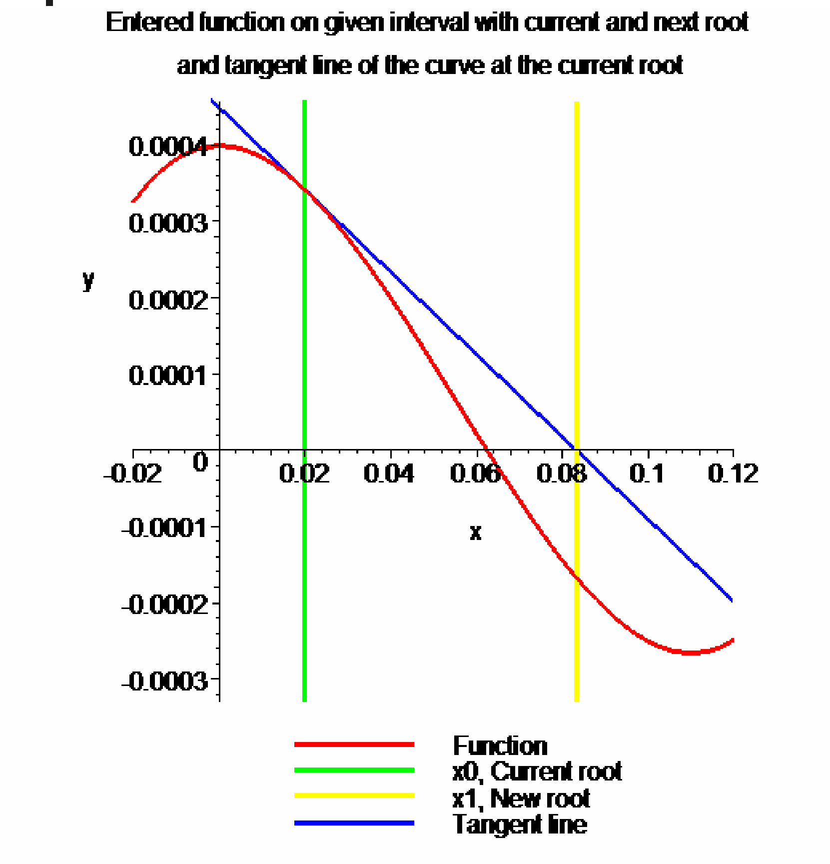


Graph of function f(x)

$$f(x) = x^3 - 0.165x^2 + 3.993\text{x}10^{-4}$$



Iteration #1



$$x_0 = 0.02$$

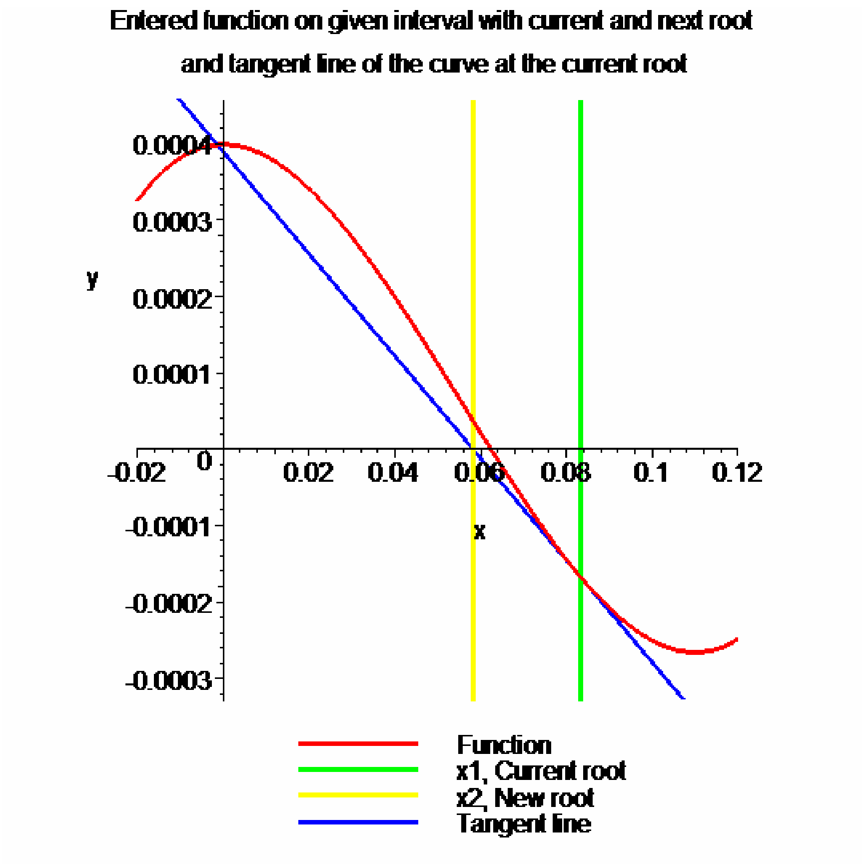
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0.02 - \frac{3.413 \times 10^{-4}}{-5.4 \times 10^{-3}}$$

$$= 0.08320$$

$$|\in_a| = 75.96\%$$

Iteration #2



$$x_1 = 0.08320$$

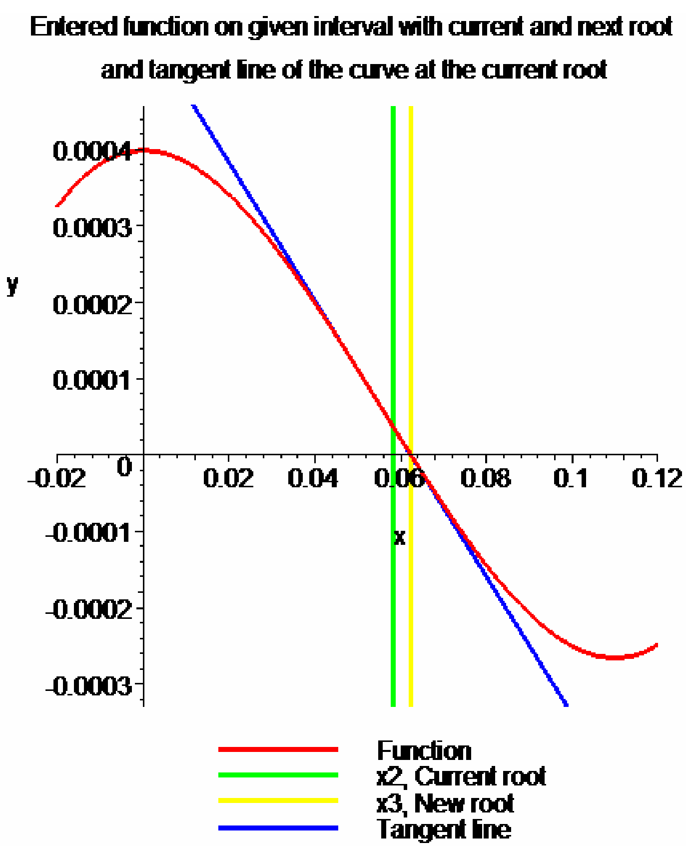
$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 0.08320 - \frac{-1.670 \times 10^{-4}}{-6.689 \times 10^{-3}}$$

$$= 0.05824$$

$$|\epsilon_a| = 42.86\%$$

Iteration #3



$$x_2 = 0.05824$$

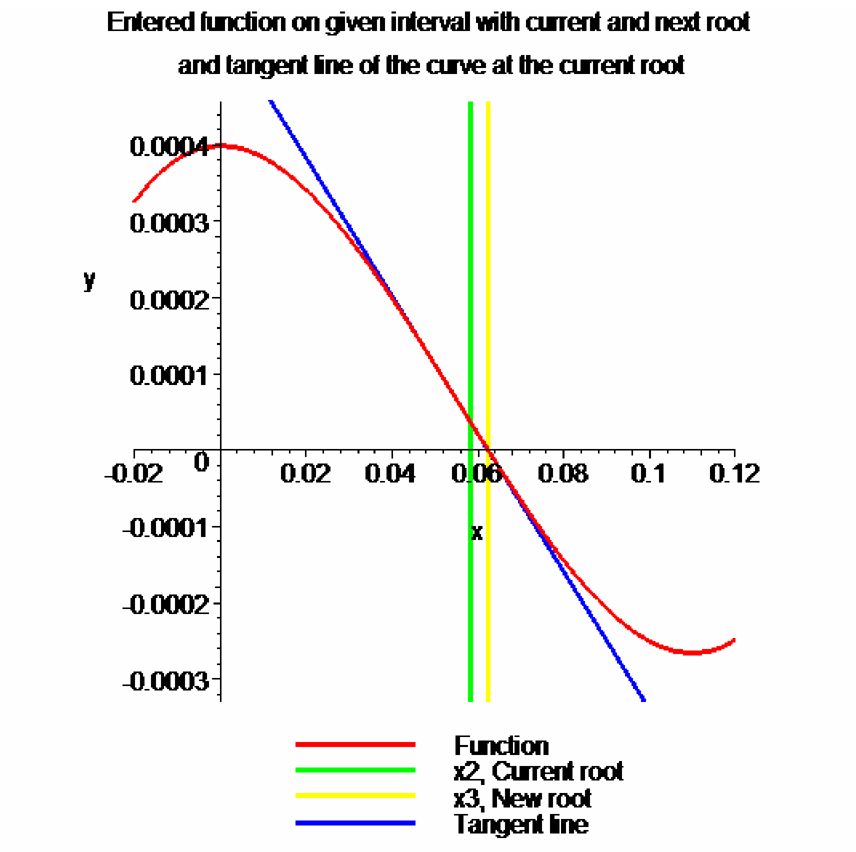
$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.05284 - \frac{3.717 \times 10^{-5}}{-9.043 \times 10^{-3}}$$

$$= 0.06235$$

$$|\in_a| = 6.592\ \%$$

Iteration #4



$$x_3 = 0.06235$$

$$x_3 = ???? - \frac{??????}{??????}$$

$$= ???? - \frac{??????}{??????}$$

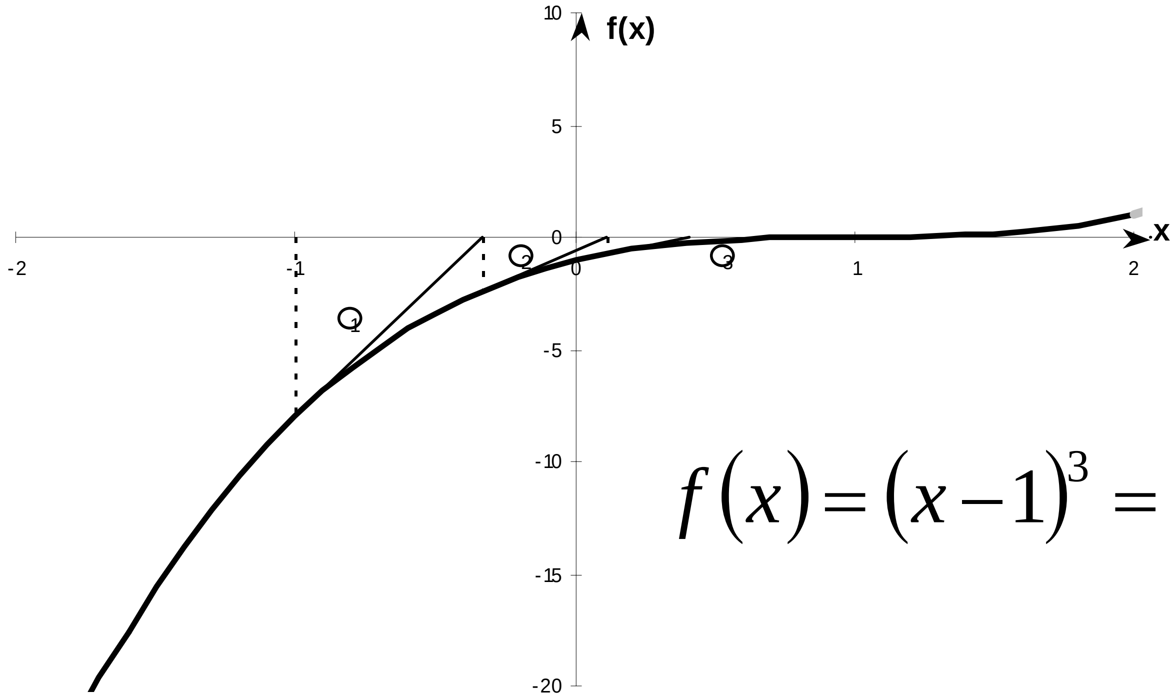
$$= ???????$$

$$\left|\in_a\right| = ?????\ \%$$

Advantages

- Converges fast, if it converges
- Requires only one guess

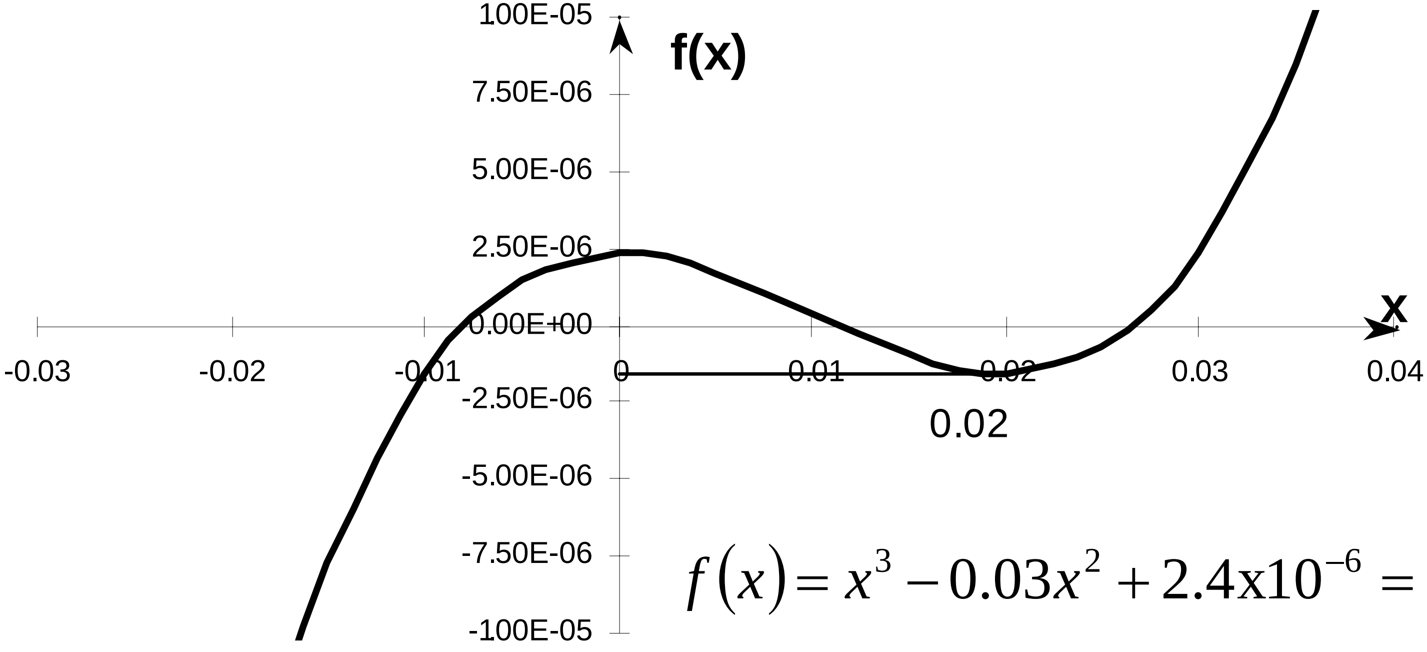
Drawbacks



$$f(x) = (x-1)^3 = 0$$

Inflection Point

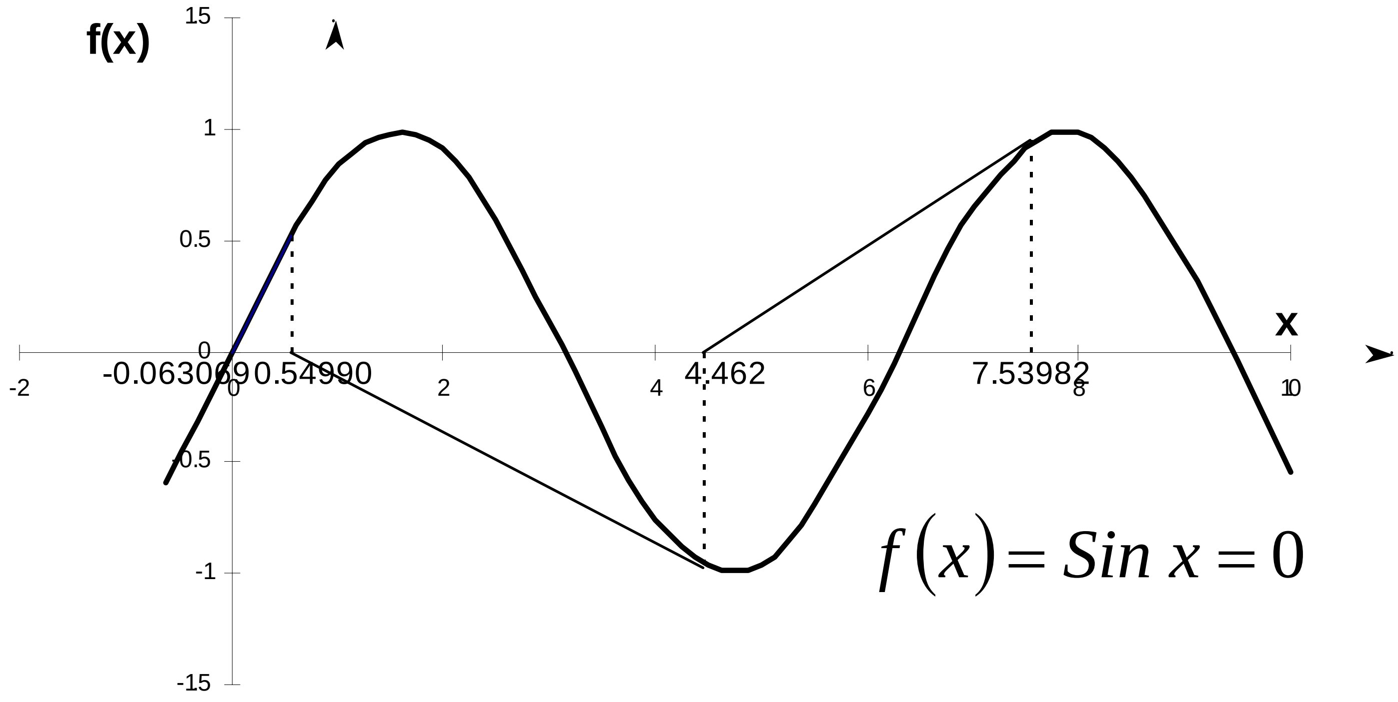
Drawbacks (continued)



$$f(x) = x^3 - 0.03x^2 + 2.4\text{x}10^{-6} = 0$$

Division by zero

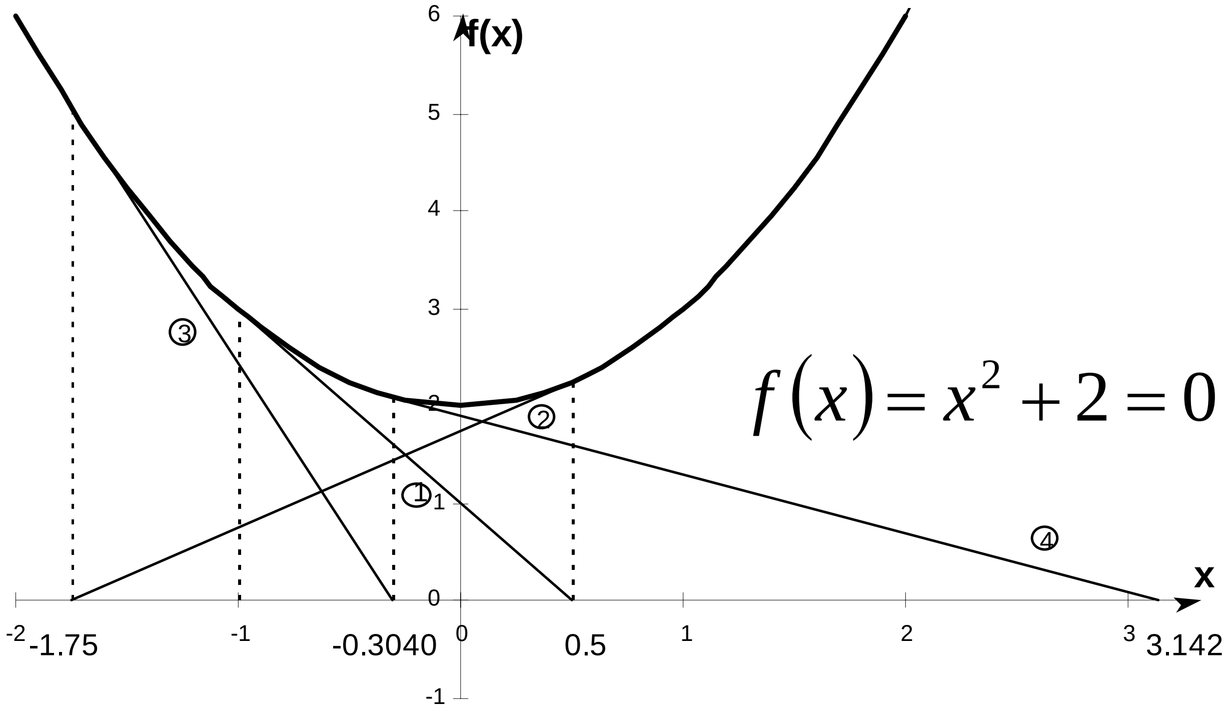
Drawbacks (continued)



$$f(x) = Sin\, x = 0$$

Root Jumping

Drawbacks (continued)



$$f(x) = x^2 + 2 = 0$$

Oscillations near Local Maxima or Minima