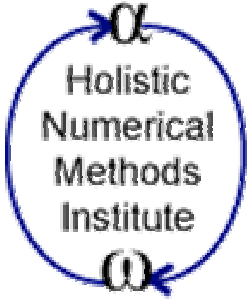
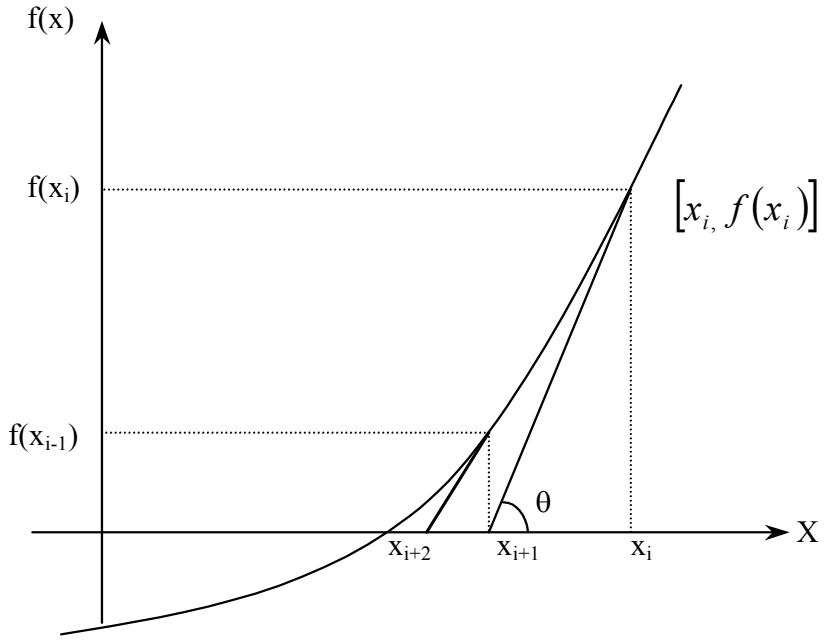


Roots of a Nonlinear Equation

Topic: Secant Method

Major: General Engineering

Secant Method



Newton's Method

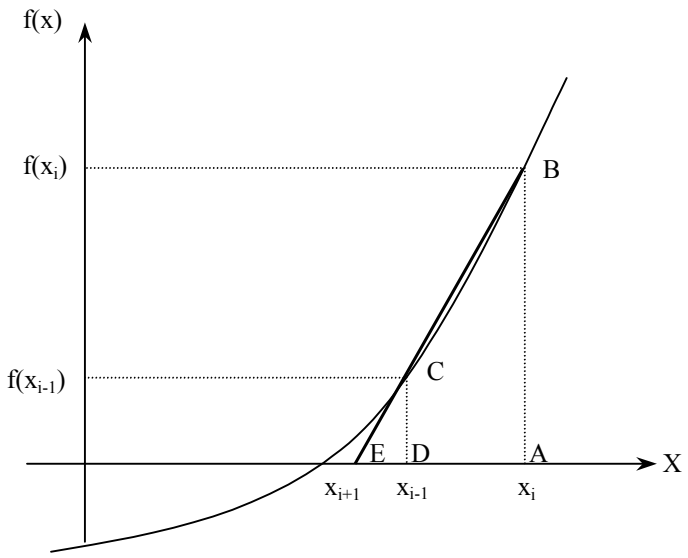
$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Approximate the derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Secant Method

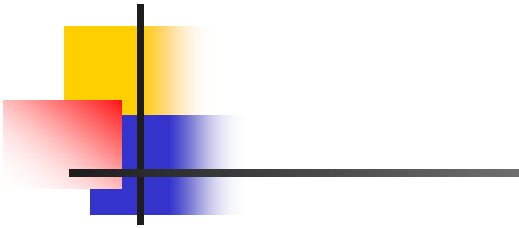


Geometric Similar Triangles

$$\frac{AB}{AE} = \frac{DC}{DE}$$

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$



Algorithm for Secant Method

Step 1

Calculate the next estimate of the root from two initial guesses

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Find the absolute relative approximate error

$$\left|\epsilon_a\right| = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \text{ x } 100$$

Step 2

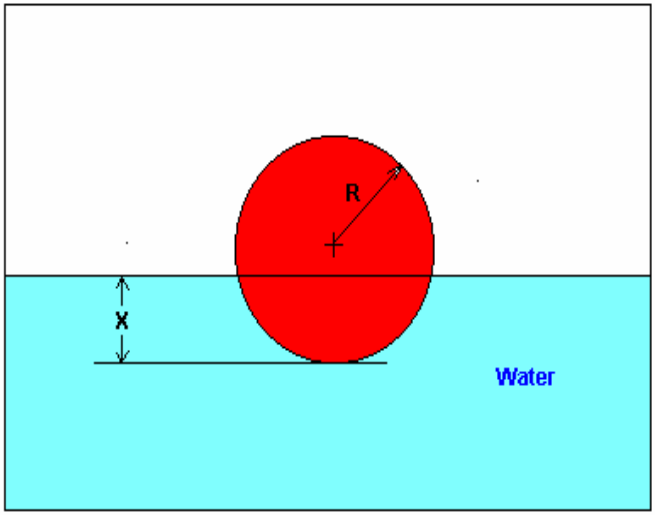
Find if the absolute relative approximate error is greater than the prespecified relative error tolerance.

If so, go back to step 1, else stop the algorithm.

Also check if the number of iterations has exceeded the maximum number of iterations.

Example

- You are working for 'DOWN THE TOILET COMPANY' that makes floats for ABC commodes. The ball has a specific gravity of 0.6 and has a radius of 1 cm. You are asked to find the distance to which the ball will get submerged when floating in water.

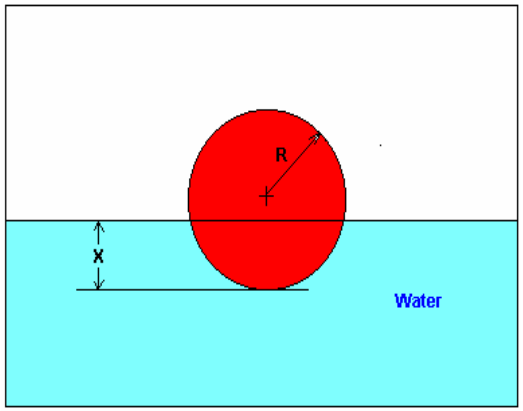


Solution

The equation that gives the depth 'x' to which the ball is submerged under water is given by

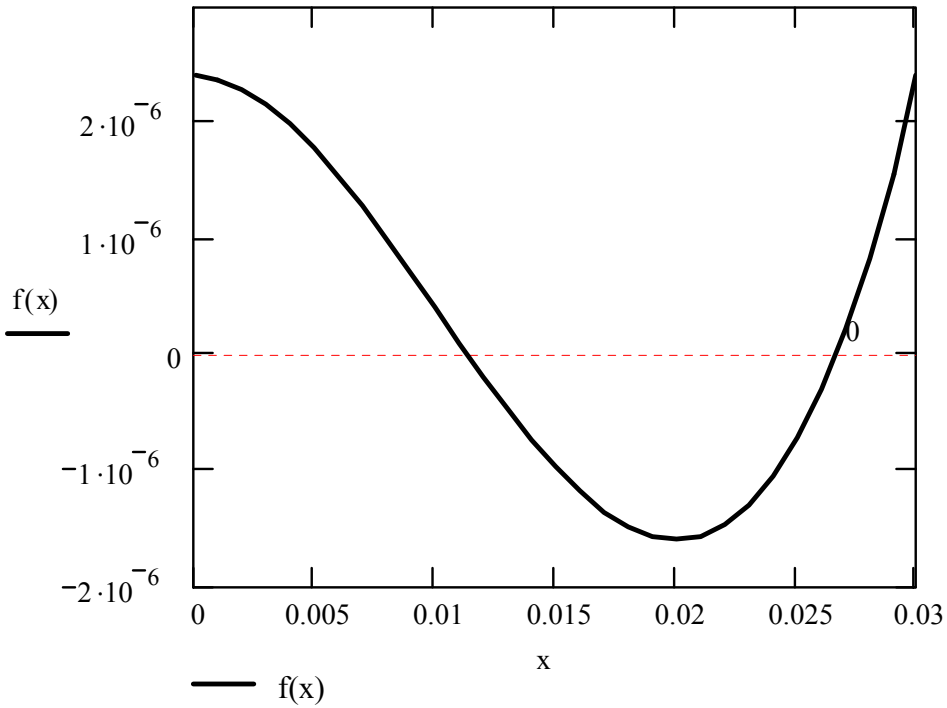
$$x^3 - 0.03x^2 + 2.4 \times 10^{-6} = 0$$

Use the Secant's method of finding roots of equations to find the depth 'x' to which the ball is submerged under water. Conduct three iterations to estimate the root of the above equation.

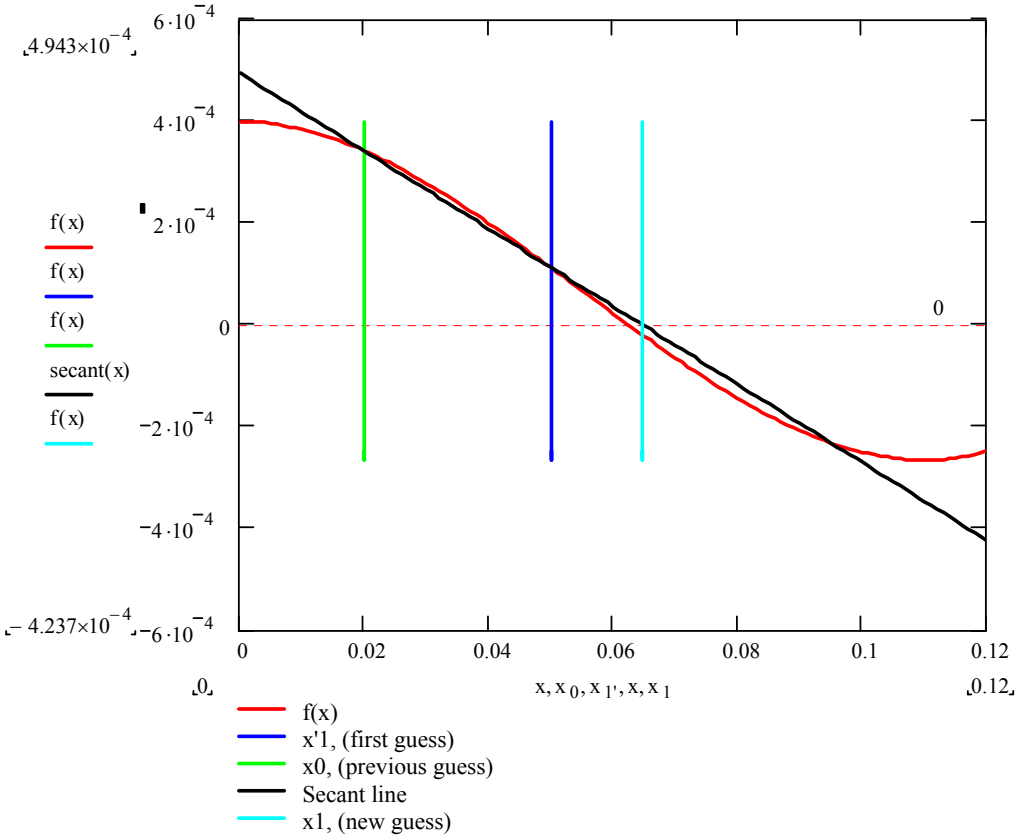


Graph of function f(x)

$$f(x) = x^3 - 0.03x^2 + 2.4\text{x}10^{-6} = 0$$



Iteration #1



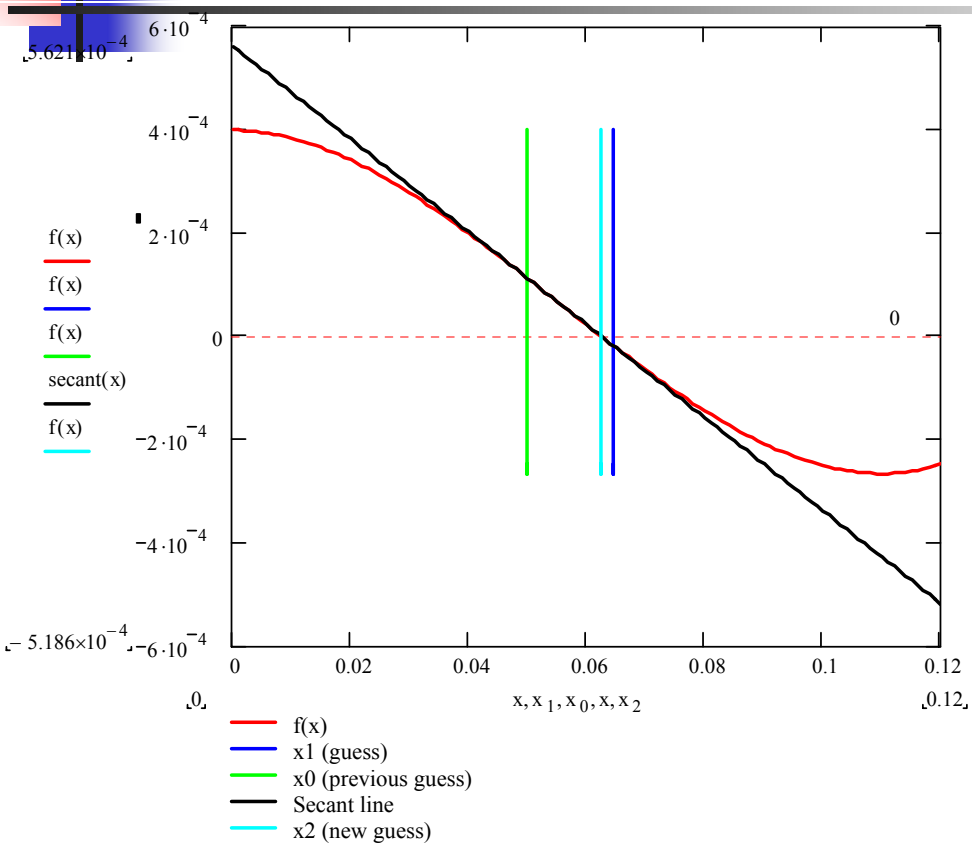
$$x_{-1} = 0.02, x_0 = 0.05$$

$$x_1 = x_0 - \frac{f(x_0)(x_0 - x_{-1})}{f(x_0) - f(x_{-1})}$$

$$x_1 = 0.06461$$

$$|\epsilon_a| = 22.61\%$$

Iteration #2



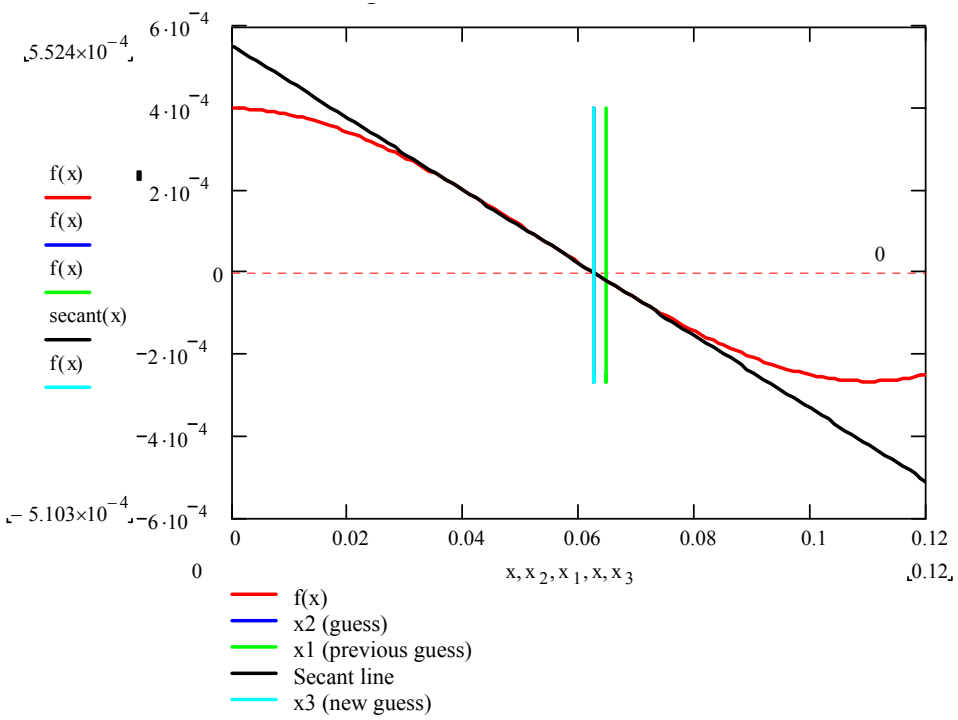
$$x_0 = 0.05, x_1 = 0.06461$$

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = 0.06241$$

$$|\epsilon_a| = 3.525\%$$

Iteration #3

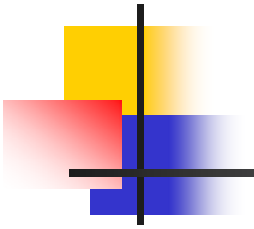


$$x_1 = 0.06461 \ , x_2 = 0.6241$$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$x_3 = 0.06238$$

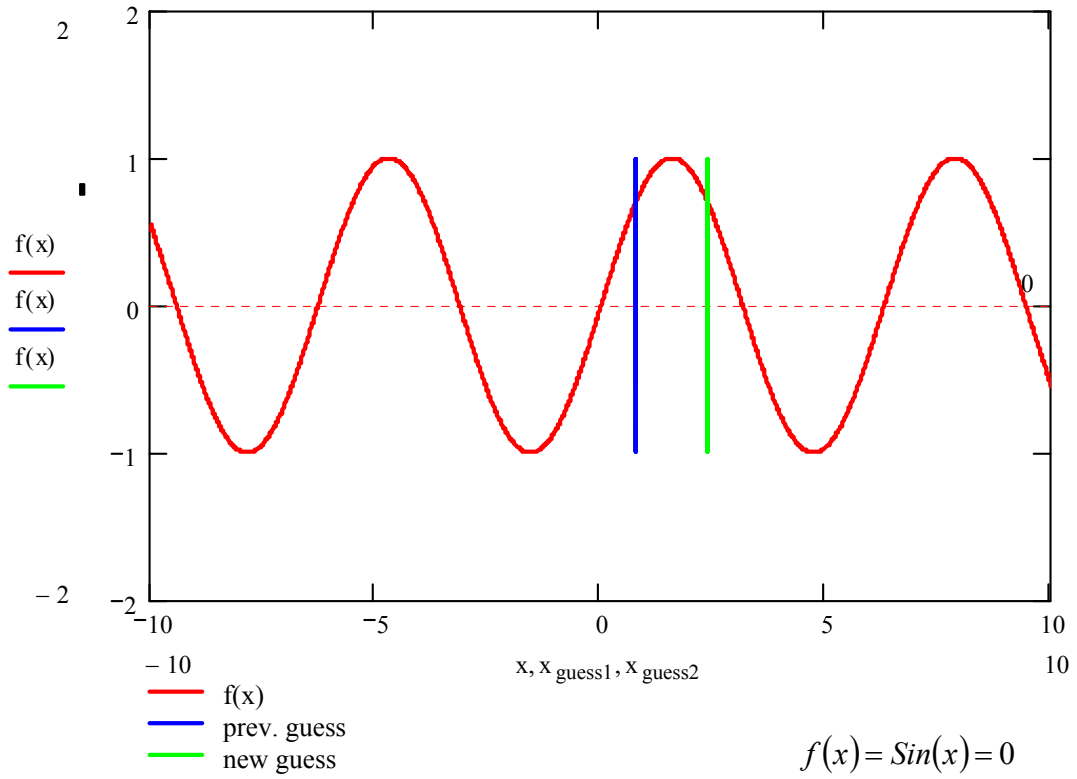
$$|\epsilon_a| = 0.0595 \ \%$$



Advantages

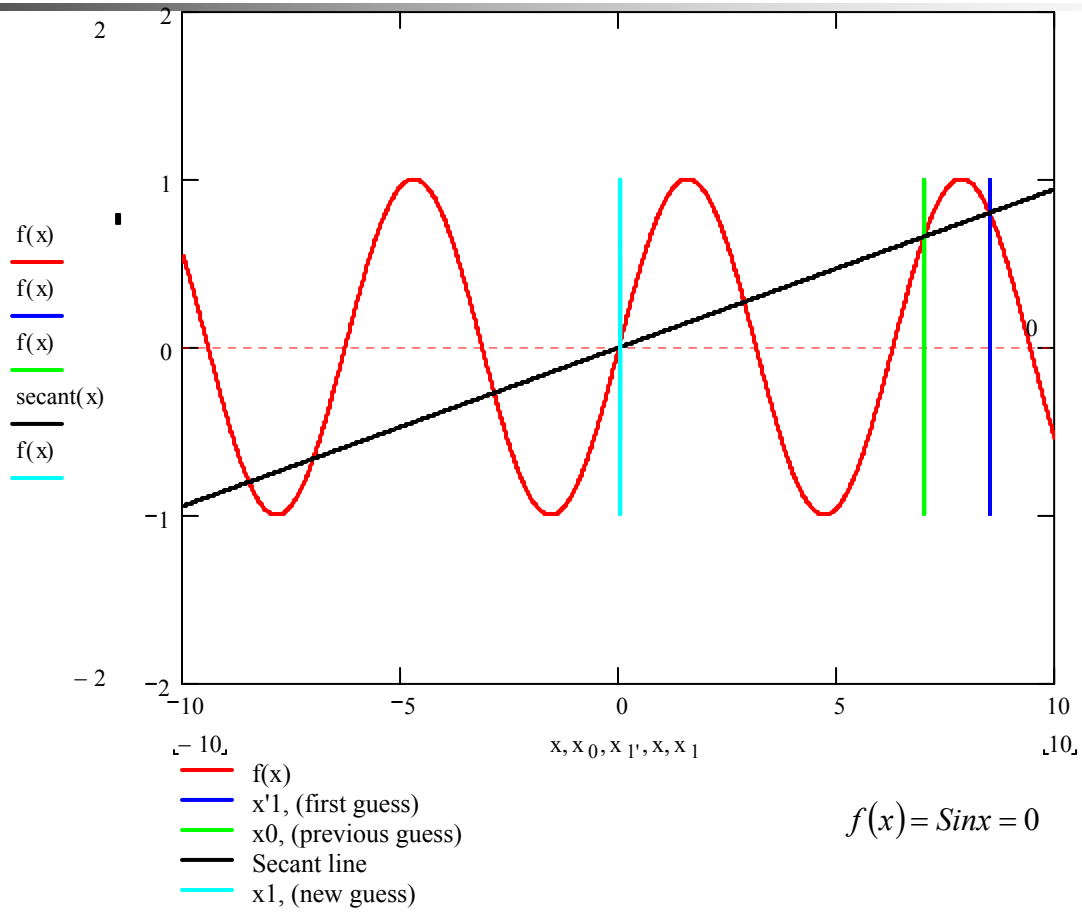
- Converges fast, if it converges
- Requires two guesses that do not need to bracket the root

Drawbacks



Division by zero

Drawbacks (continued)



$$f(x) = Sinx = 0$$

Root Jumping