

Chapter 08.00F

Physical Problem for Industrial Engineering Ordinary Differential Equations

Speed control of DC motors

In this example we will discuss the closed loop speed control of a DC motor. Figure 1 shows three different DC motors and Figure 2 depicts the inside of a DC motor. Universal Motors which are essentially DC motors are widely used in applications where the speed of a process needs to be controlled. Such applications are encountered frequently in our daily lives such as controlling the speed of fans or controlling the speed of hand-held tools such as drills, and in industrial automation applications such as controlling the speed of conveyor belts.



Figure 1 A variety of DC motors

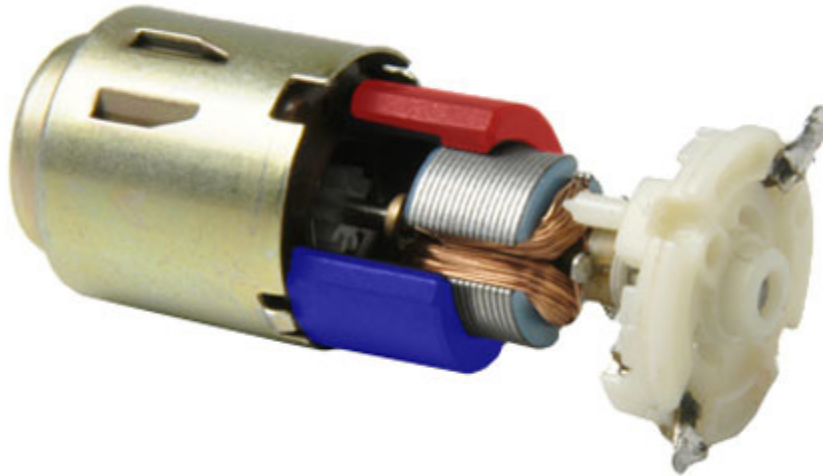


Figure 2 The inside of a DC motor

Before we can discuss the speed control of a DC motor, it is important to understand the physical time varying relationships that determine the operating characteristics of a DC motor.

Equation (1) below describes the linear relationship between the torque T and the current i_a applied to the motor. The slope of the line is the torque constant K_t . Equation (2) describes the linear relationship between the back emf E_b and the armature speed w . The slope of the line K_b is the voltage constant. Equation (3) describes the components of the armature voltage V as sum of the back emf and the voltage drop across armature resistance Ri_a . Finally, Equation (4) describes the relationship between torque, acceleration and speed of the motor in a no-load system as the sum of the angular acceleration dw/dt multiplied by the inertia of the motor and the load, and the damping of the system f multiplied by the armature speed.

$$T(t) = K_t i_a(t) \quad (1)$$

$$E_b(t) = K_b w(t) \quad (2)$$

$$V(t) = Ri_a(t) + E_b(t) \quad (3)$$

$$T(t) = I \frac{dw}{dt} + fw(t) \quad (4)$$

The equation for the speed of the motor in relation to the voltage input can be derived from these relationships as follows. Substitute the value of back emf from Equation (2) into Equation (3) as:

$$V(t) = Ri_a(t) + K_B w(t) \quad (5)$$

Equating Equations (1) and (4) and solving for i_a results in

$$i_a = \frac{I \frac{dw}{dt} + fw(t)}{K_T} \quad (6)$$

Finally substitute the value for i_a from Equation (6) into Equation (5) as

$$K_T V(t) = RI \frac{dw}{dt} + (Rf + K_T K_B)w(t) \quad (7)$$

Equation (7) is a first order differential equation which describes the open-loop response of the motor to a voltage input where the output variable system (speed of the motor) is not considered in the control mechanism. In the next section we will consider the closed-loop control of the motor.

Closed loop control

Figure 3 illustrates the block diagram of a proportional control system used to control the speed of a DC motor where K_P is the proportional gain. The “C” block is the summing point which generates an error term e equal to the difference between the desired speed set by the operator (w_d) and the actual speed of the motor. The “DC Motor” block transforms the amplified error input from the “Control Action” block to the output speed of the motor.

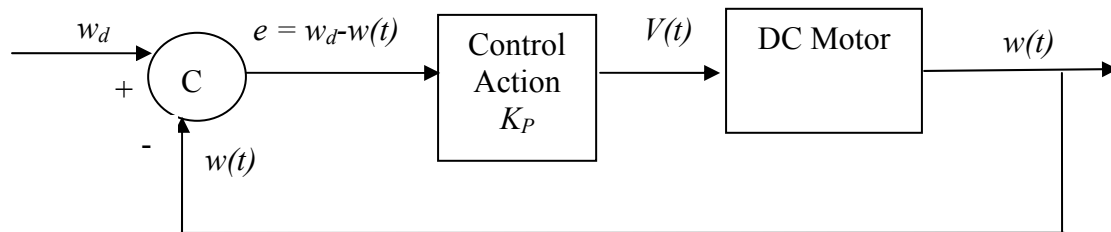


Figure 3 Block diagram of a DC motor control systems

Based on the control system block diagram

$$V(t) = K_P [w_d - w(t)] \quad (8)$$

Substituting the value of $V(t)$ from Equation (8) in to Equation (7) and rearranging terms results in the differential Equation (9) which describes the relationship of the actual speed of the motor to the desired speed.

$$K_T K_P w_d = RI \frac{dw}{dt} + [Rf + K_T (K_B + K_P)] w(t) \quad (9)$$

Example application

Consider a machine vision quality inspection system which inspects parts for defects shown in Figure 4. The parts are positioned in fixtures on a conveyor driven by a DC motor. The conveyor is required to ramp up to a certain speed within a specific time (before a part enters the field of view of the camera), maintain a constant speed while the part is under the camera and finally come to a stop once the inspection of the part is completed. Many such applications which utilize DC motors have requirements associated with the steady-state speed and/or a ramp up time for the motor to reach this speed.

Consider a DC motor which has the following specifications:

Voltage constant $K_B = 0.06 \text{ V}\cdot\text{s}/\text{rad}$

Torque constant $K_T = 0.06 \text{ N}\cdot\text{m}/\text{A}$

Armature resistance $R = 2 \Omega$

Moment of inertia $I = 6 \times 10^{-4} \text{ N}\cdot\text{m}\cdot\text{s}^2/\text{rad}$

The open loop response of the motor to a voltage input using Equation (7) and assuming a system without damping ($f = 0 \text{ N}\cdot\text{m}\cdot\text{s}/\text{rad}$) is

$$V(t) = (0.02) \frac{dw}{dt} + (0.06)w(t).$$

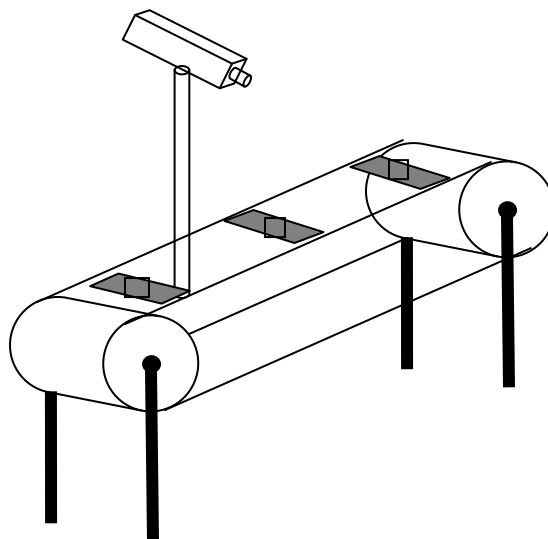


Figure 4 Machine vision system schematic

The speed of the motor $w(t)$ can therefore be controlled by changing the magnitude of this input voltage ($v(t)$). Assuming the initial condition $w(0) = 0$ and a specific input voltage, the above ordinary differential equation can be solved to determine the steady-state speed of the motor as well as the time it takes for the motor to ramp up to this speed. The information obtained from the solution is important in selection of the appropriate voltage input and/or motor for a particular application to assure that the motor response time and speed is sufficient for the task.

Similarly, the design of a closed loop controller to control the speed of the motor requires the solution to the ordinary differential Equation (9) to determine the user settings to obtain the necessary output from the motor. This application is posed as Exercise Problem 5 below.

Exercise problems

For the DC motor in the Example Problem, answer the following questions:

1. Draw the speed response graph $w(t)$ vs t for a step input of 20 Volts without the damping of the system ($f = 0$ N·m·s /rad)
2. Draw the speed response graph for a step input of 20 Volts considering the damping of the system where $f = 1.0 \times 10^{-4}$ N·m·s/rad .
3. What is the difference between the steady-state speed of the motor with and without damping?
4. Consider a closed loop control system with gain $K_p = 10$. What is the closed loop speed response graph of motor to a desired setting of 100 rad/s?
5. The answer to Question#4 is less than 100 rad/s due to the steady state error present in first order systems. What should be the desired speed setting for the motor speed response to be 100 rad/s?

Additional reading

- [1] Boucher, T.O., Computer Automation in Manufacturing: An Introduction, Chapman & Hall, London, 1996.
- [2] Bateson, R.N., Introduction to Control System Technology, 6th Edition, Prentice Hall, New Jersey, 1999.

ORDINARY DIFFERENTIAL EQUATIONS

Topic	Ordinary Differential Equations
Summary	Many appliances used in our daily lives as well as numerous applications in industrial automation require controlling speed of a DC motor to function properly. Classical control theory requires solving differential equations to control these types of motors.
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