Chapter 11.00E

Physical Problem for Fast Fourier Transform Industrial Engineering

Problem Statement

Forecasting is a very important tool we use in daily life. We forecast traffic or length of the line at the local coffee shop based on the time of day. Figure 1 below shows monthly net electricity power generation (Billion Kilowatt-hours) for years 1973, 83, 93 and 2003. This time series exhibits interesting properties as it has both an upward trend but also seasonal behavior. Therefore, traditional methods used for forecasting stationary time series (moving averages and exponential smoothing), trend based methods (regression), or methods used for seasonal series would introduce significant errors if used for forecasting the net electricity power generation for year 2023.

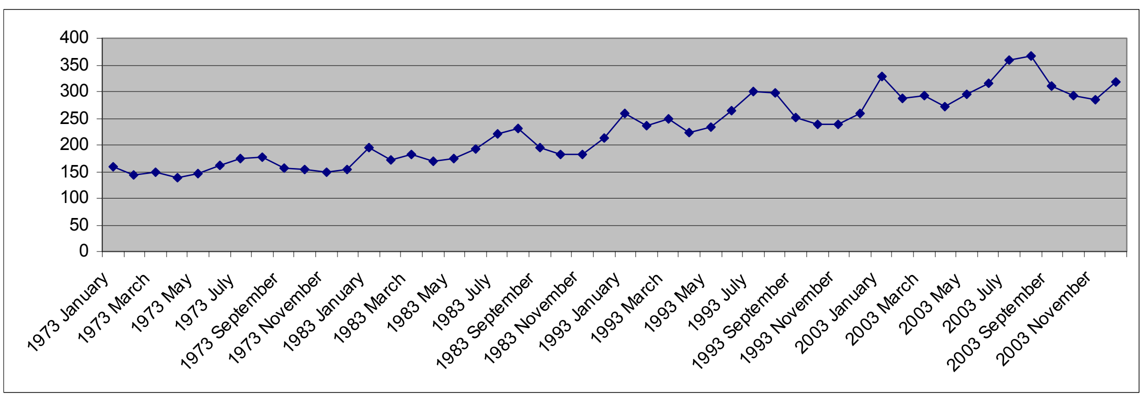

Figure 1 Net electricity power generation.

Forecasting also plays a key role in the operations of firms. For example demand forecasts are used in capacity planning, inventory control and raw material procurement. Although a certain portion of future demand is random and unpredictable, other portions may exhibit trends, cycles and seasonal variations which may help forecast values reasonably close to the actual demand values.

Modeling Seasonal Demand

Seasonal demand is characterized by a pattern that repeats every N periods. Figure 2 shows a typical seasonal demand pattern where the length of a season is shown with N. Most seasonal demand forecasting models require that N is specified a priori. The most common method of seasonal forecasting is to use seasonal multipliers where the multiplier represents the difference from the average demand. For example a multiplier of 0.5 in a period means that the demand in that particular period is half of the average demand over the N period season.

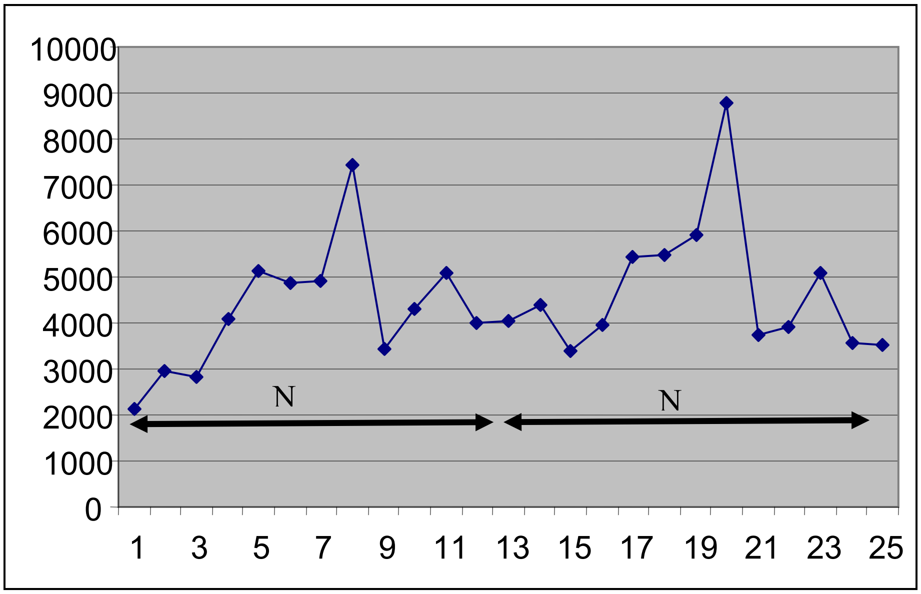

Figure 2 Typical seasonal demand pattern.

Trigonometric models of Fourier transforms can also be used to forecast demand for products which exhibit periodicity. Snow tires, antifreeze and air conditioners are examples of such products. Consider the data in Table 1 where the sale of a retail outlet for 3 styles of clothing A, B and C between May 4th and April 7th is shown. The same data is plotted in Figure 3 to better visualize the demand behavior which shows a periodic change.

Table 1 Sales of a retail outlet.

	2004/05	2004/06	2004/07	2004/08	2004/09	2004/10	2004/11	2004/12	2005/01	2005/02	2005/03	2005/04
A	2112	2952	2831	4095	5118	4848	4931	7437	3451	4286	5107	3990
B	9410	9939	7327	9354	12760	13331	13913	20031	9238	11402	13673	10469
C	2272	3136	2814	4040	5331	5395	5547	7795	3932	4548	5319	3877

	2005/05	2005/06	2005/07	2005/08	2005/09	2005/10	2005/11	2005/12	2006/01	2006/02	2006/03	2006/04
A	4058	4373	3403	3944	5432	5488	5894	8761	3733	3934	5102	3549
B	10203	10681	7466	9019	12910	14003	14761	21419	10327	11524	14034	10044
C	3947	4075	3064	3930	5195	5526	5873	8308	4132	4539	5647	3939

	2006/05	2006/06	2006/07	2006/08	2006/09	2006/10	2006/11	2006/12	2007/01	2007/02	2007/03	2007/04
A	3507	4329	3352	4110	4959	4537	4938	7295	3149	3458	3899	2974

B	9801	10137	7087	8567	12072	11824	12633	19097	9098	10346	11387	8707
C	3733	4667	3411	4234	5598	5270	5218	7715	3833	3931	4382	3437

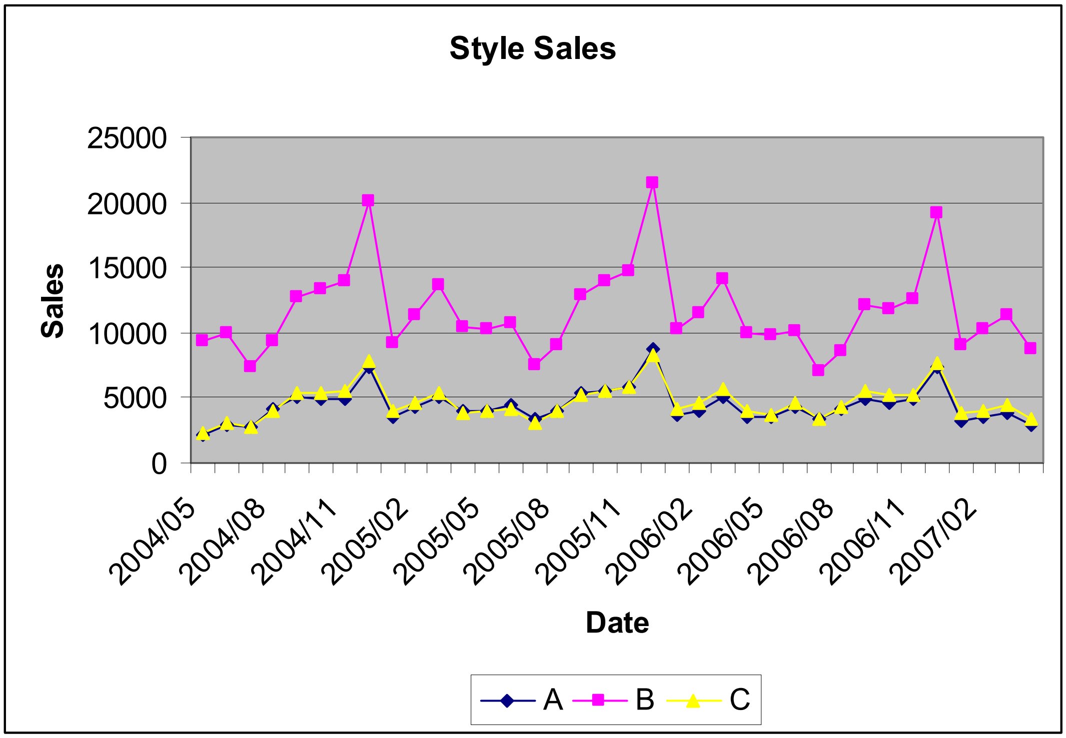

Figure 3 Plot of the sales data in Table 2

The shape of the data suggests the use of a function of the form

$$\hat{D}_t = A + B\sin\frac{2\pi t}{N} \quad (1)$$

where A is the origin of the sine wave, B is the amplitude and $2\pi/N$ is the angular frequency. For the above problem, the seasonal cycle repeats every 12 months, therefore Equation (1) can be stated as

$$\hat{D}_t = A + B\sin\frac{\pi t}{6}$$

The sum of square of errors the can be written as

$$SS_E = \sum_{t=1}^{N}[\hat{A} + \hat{B}\sin\frac{\pi t}{6} - x_t]^2 . \quad (2)$$

Taking the partial derivatives of Equation (2) with respect to $\hat{A}$ and $\hat{B}$ respectively, we obtain the following equations.

$$\frac{\partial SS_E}{\partial \hat{A}} = 2\sum_{t=1}^{N}[\hat{A} + \hat{B}\sin\frac{\pi t}{6} - x_t] = 0 \quad (3)$$

$$\frac{\partial SS_E}{\partial \hat{B}} = 2\sum_{t=1}^{N}[\hat{A} + \hat{B}\sin\frac{\pi t}{6} - x_t]\sin\frac{\pi t}{6} = 0 \quad (4)$$

The parameters $\hat{A}$ and $\hat{B}$ can now be determined by solving Equations (3) and (4) simultaneously.

Questions

a) Using the values for style A between 2004/05 and 2006/04, forecast the sales values for 2006/05 to 2007/04.
b) Solve Question 1 for Styles B and C.
c) What are your significant sources of error when your forecasts are compared with the actual sales figures?

Topic	FAST FOURIER TRANSFORM
Sub Topic	Physical Problem
Summary	Seasonal Demand Forecasting
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Last Revised	August 16, 2009
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