

# Roots of a Nonlinear Equation



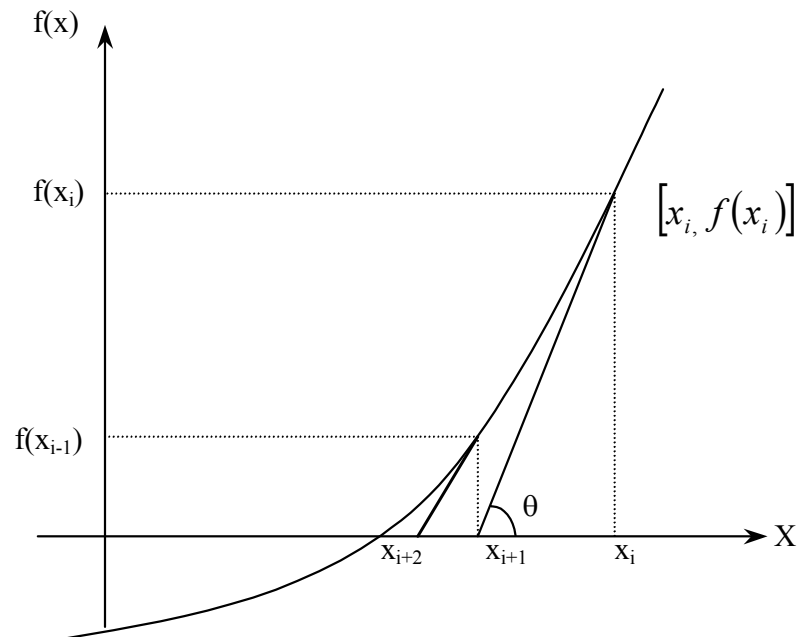
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Topic: Newton-Raphson Method

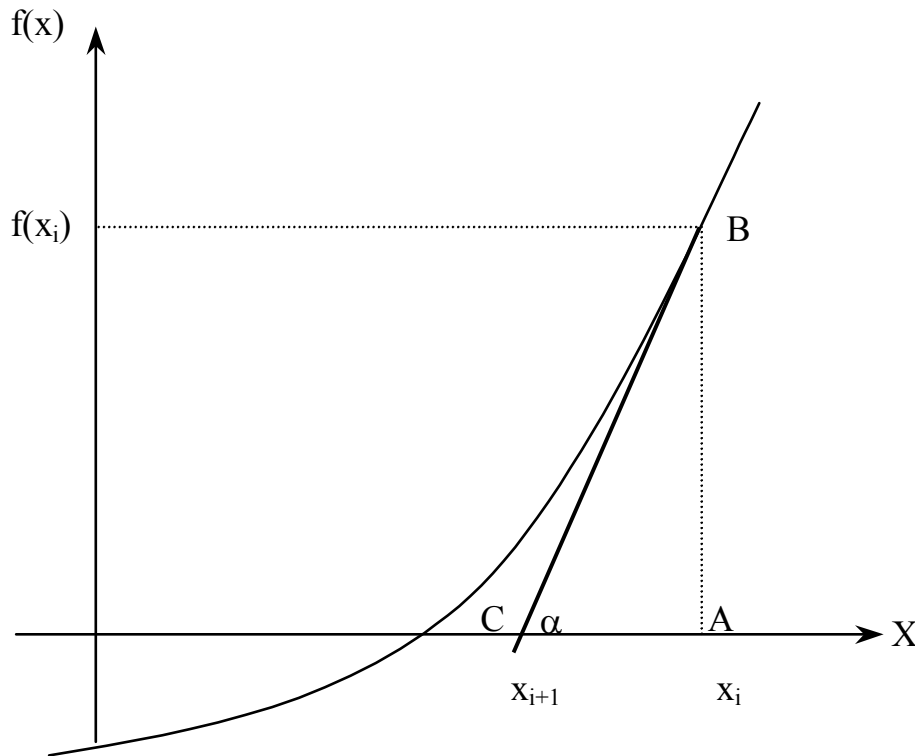
Major: Chemical Engineering

# Newton-Raphson Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$



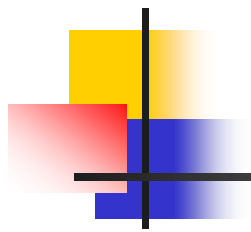
# Derivation



$$\tan(\alpha) = \frac{AB}{AC}$$

$$f'(x_i) = \frac{f(x_i)}{x_i - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$



# Algorithm for Newton-Raphson Method



# Step 1

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Evaluate  $f'(x)$  symbolically



## Step 2

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Calculate the next estimate of the root

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Find the absolute relative approximate error

$$|\epsilon_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$$



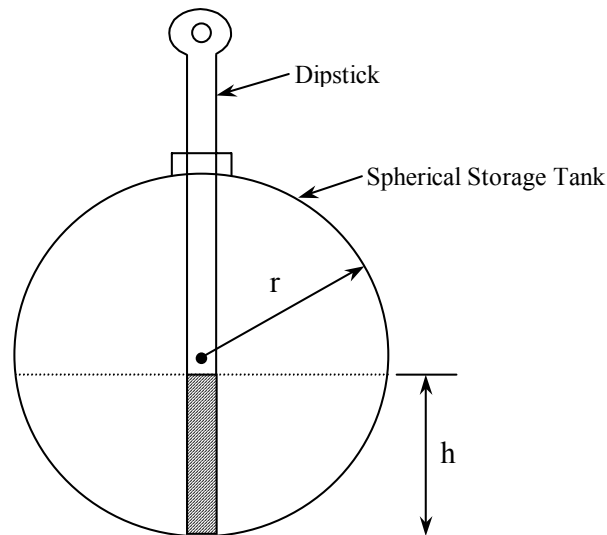
## Step 3

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- Find if the absolute relative approximate error is greater than the pre-specified relative error tolerance.
- If so, go back to step 2, else stop the algorithm.
- Also check if the number of iterations has exceeded the maximum number of iterations.

# Example

- You have a spherical storage tank containing oil. The tank has a diameter of 6 ft. You are asked to calculate the height 'x' to which a dipstick 8 ft long would be wet with oil when immersed in the tank when it contains 4 ft<sup>3</sup> of oil.

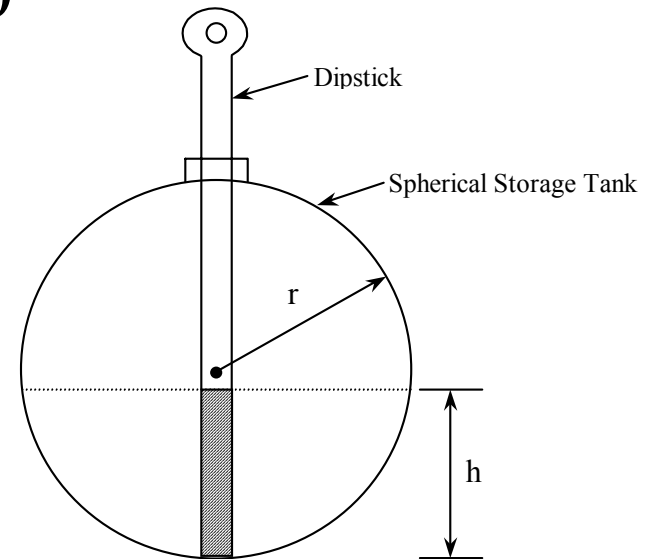


# Solution

The equation that gives the height 'h' of liquid in the spherical tank for the given volume and radius is given by:

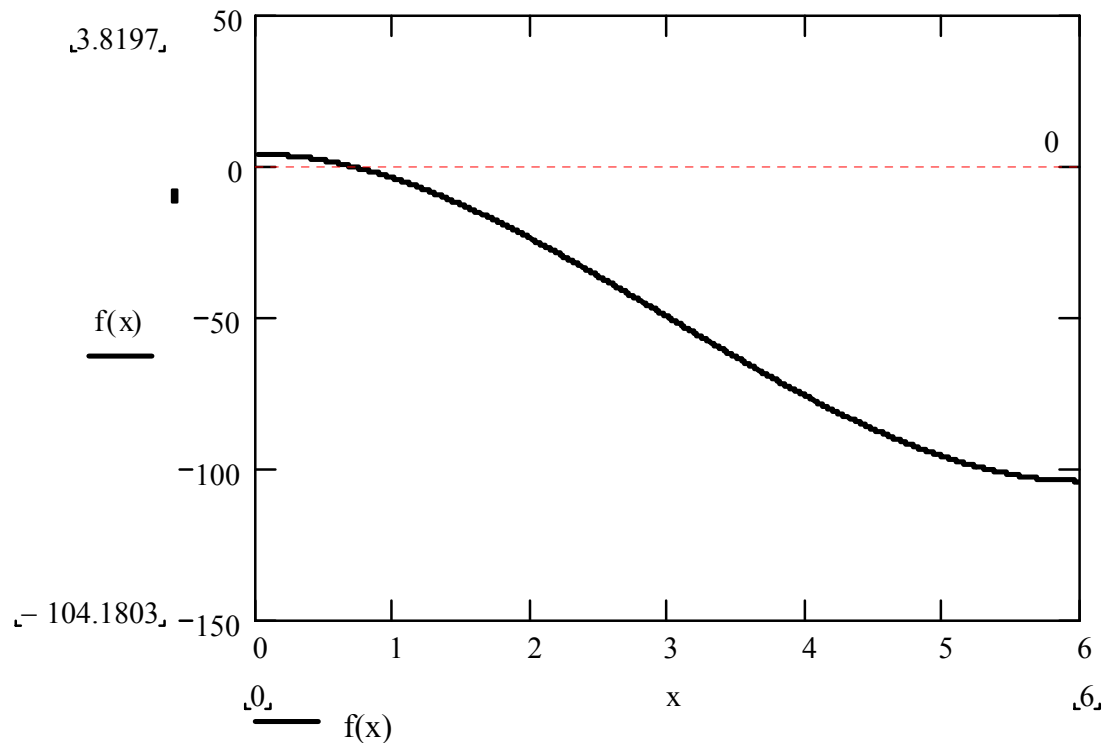
$$f(h) = h^3 - 9h^2 + 3.8197 = 0$$

Use the Newton method of finding roots of equations to find the depth 'h' to which the ball is submerged under water. Conduct three iterations to estimate the root of the above equation.

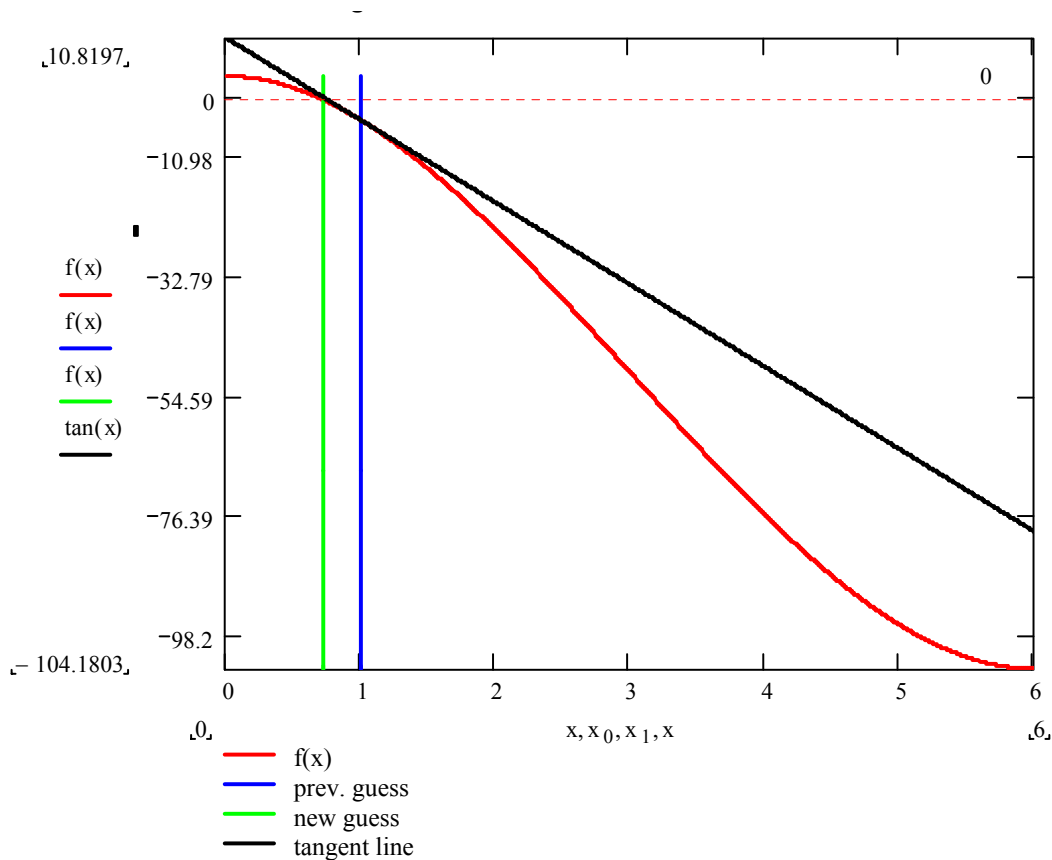


# Graph of function $f(x)$

$$f(h) = h^3 - 9h^2 + 3.8197 = 0$$



# Iteration #1



$$h_0 = 1$$

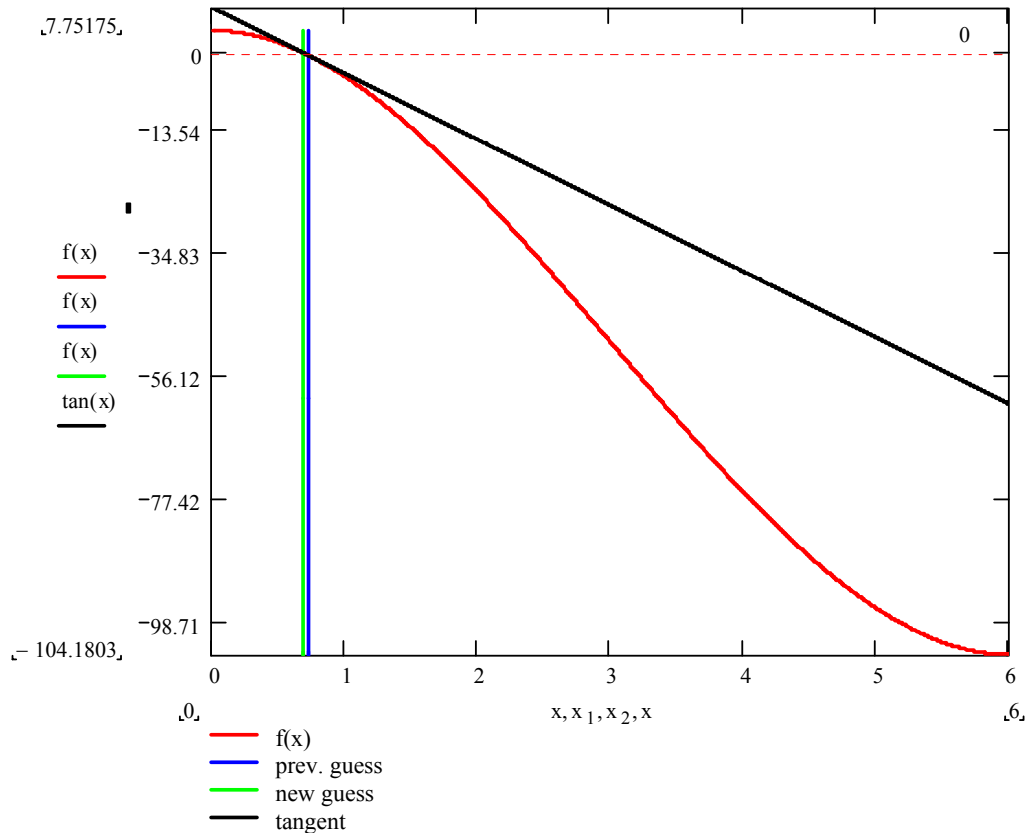
$$h_1 = h_0 - \frac{f(h_0)}{f'(h_0)}$$

$$h_1 = 1 - \frac{-4.1803}{-15}$$

$$= 0.7213$$

$$|\epsilon_a| = 38.64\%$$

# Iteration #2



$$h_1 = 0.7213$$

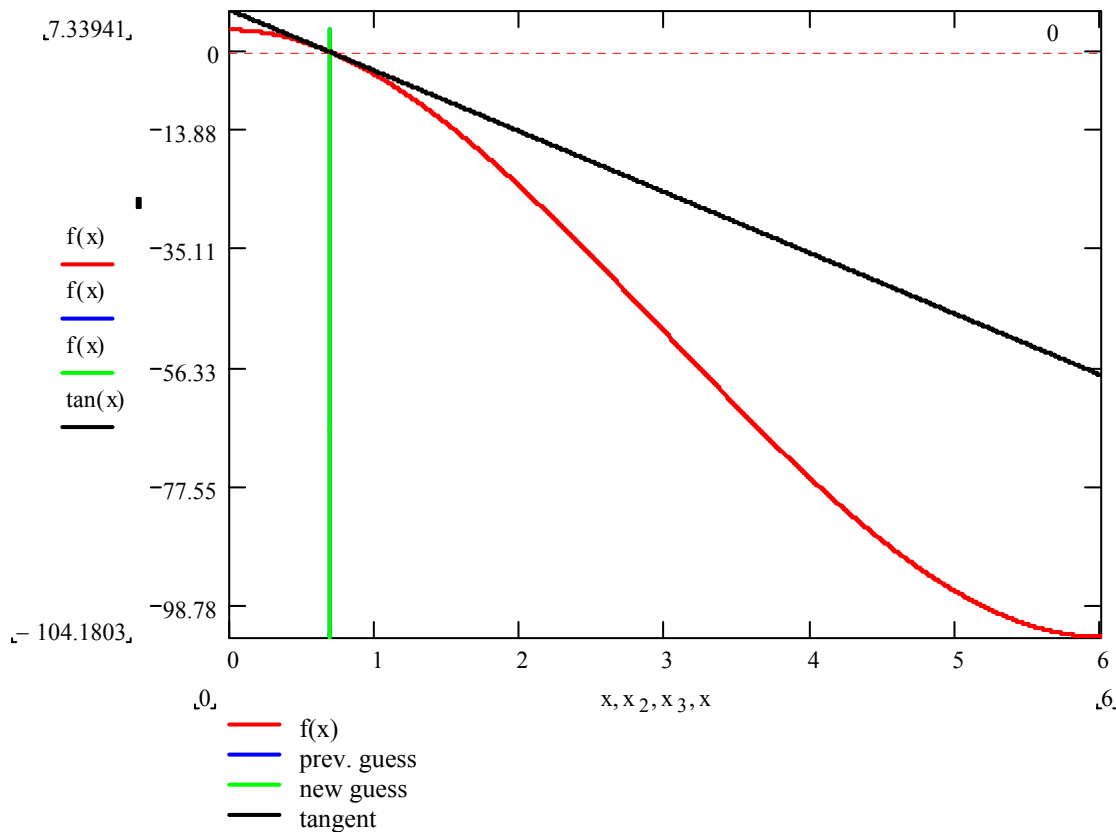
$$h_2 = h_1 - \frac{f(h_1)}{f'(h_1)}$$

$$h_2 = 0.7213 - \frac{-0.4875}{-11.423}$$

$$= 0.6786$$

$$|\epsilon_a| = 6.29\%$$

# Iteration #3



$$h_2 = 0.6786$$

$$h_3 = h_2 - \frac{f(h_2)}{f'(h_2)}$$

$$= 0.6786 - \frac{-0.01228}{-10.833}$$

$$= 0.6774$$

$$|\epsilon_a| = 0.1771 \%$$

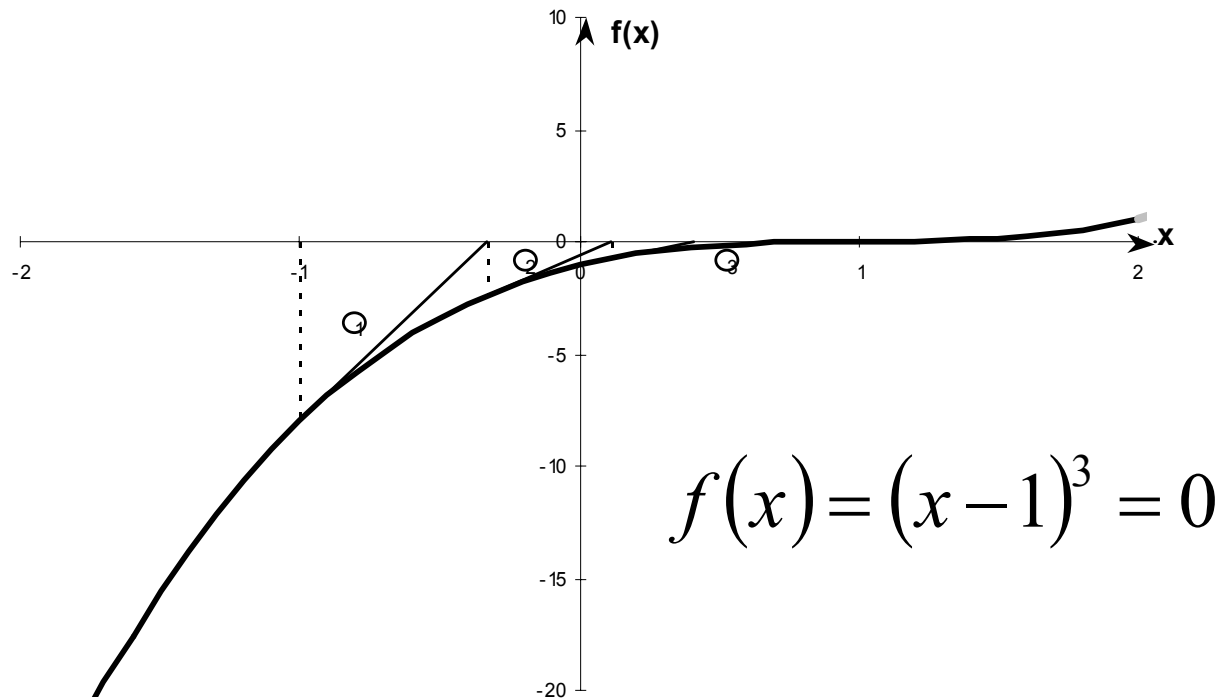


# Advantages

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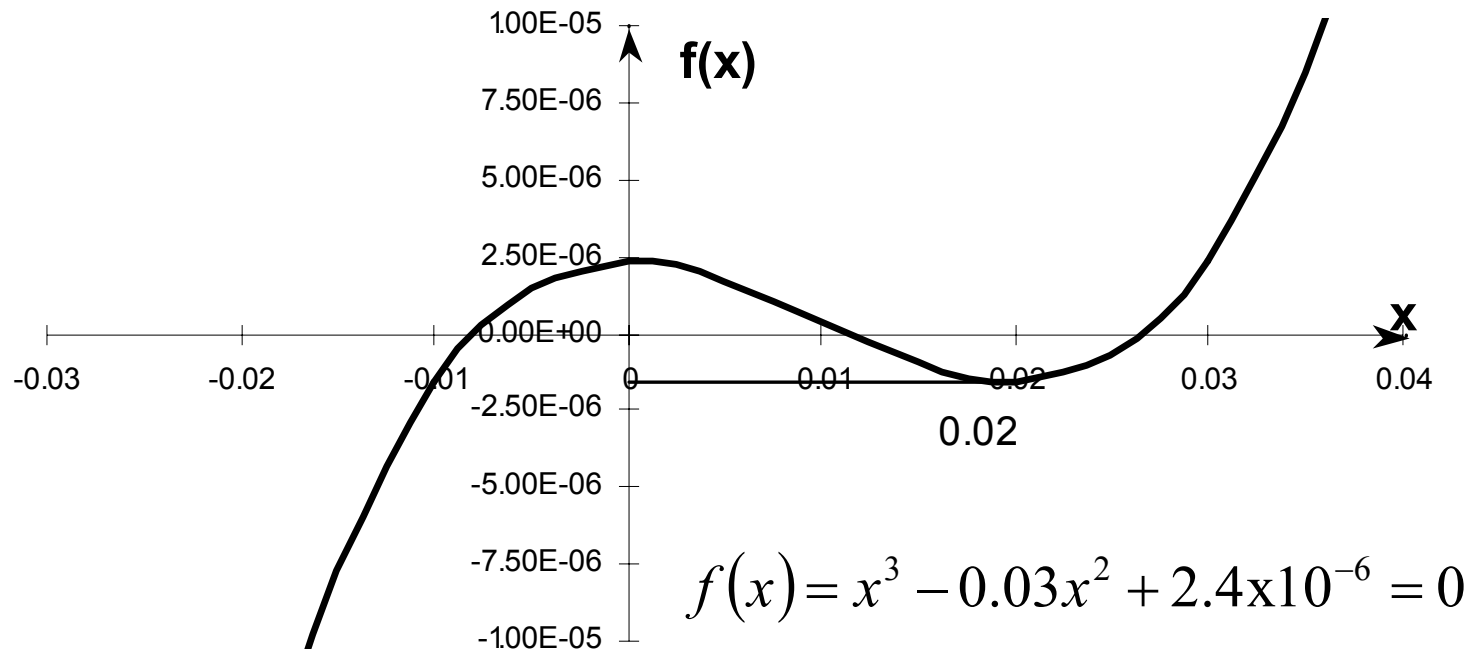
- Converges fast, if it converges
- Requires only one guess

# Drawbacks



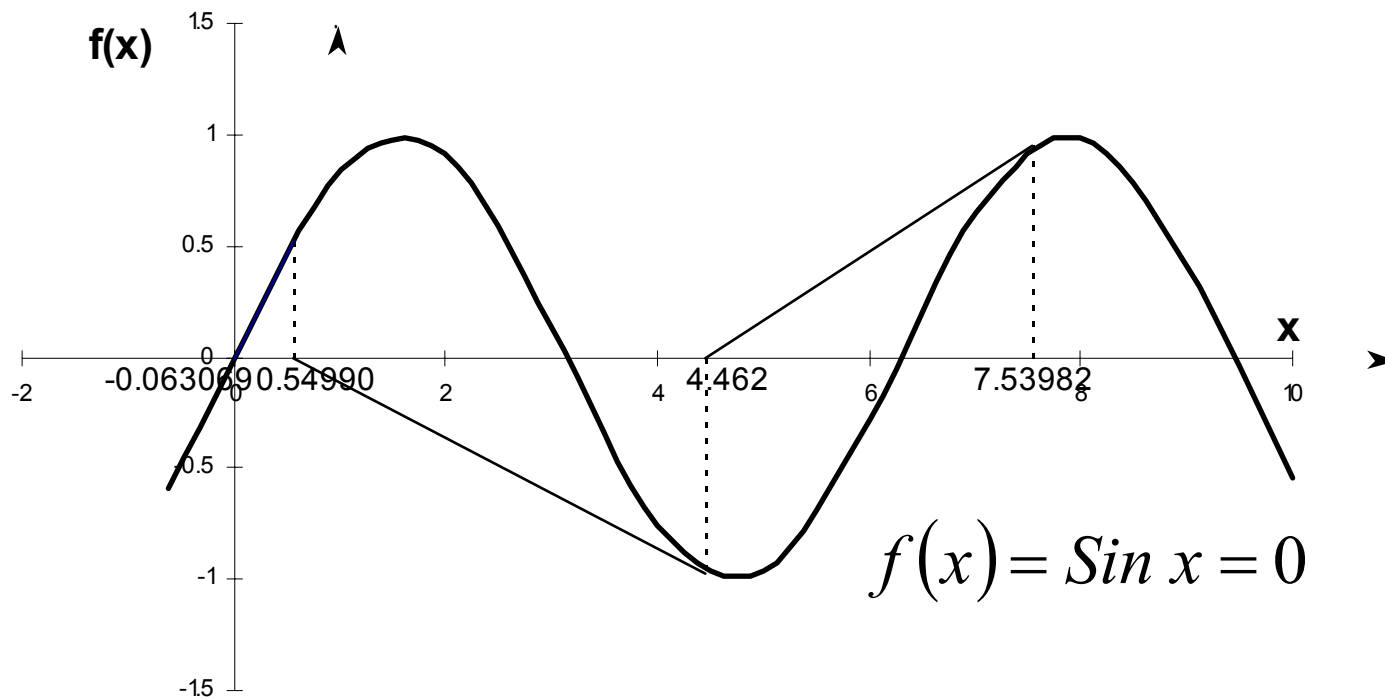
Inflection Point

# Drawbacks (continued)



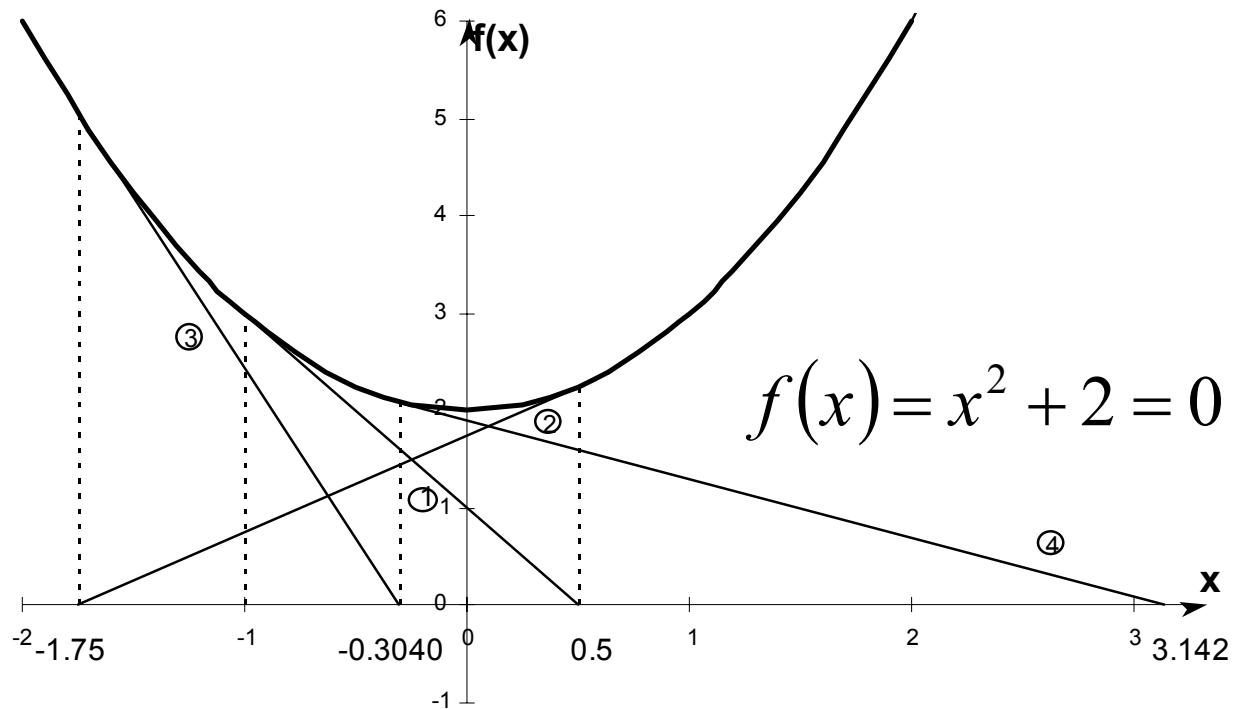
Division by zero

# Drawbacks (continued)



Root Jumping

# Drawbacks (continued)



Oscillations near Local Maxima or Minima