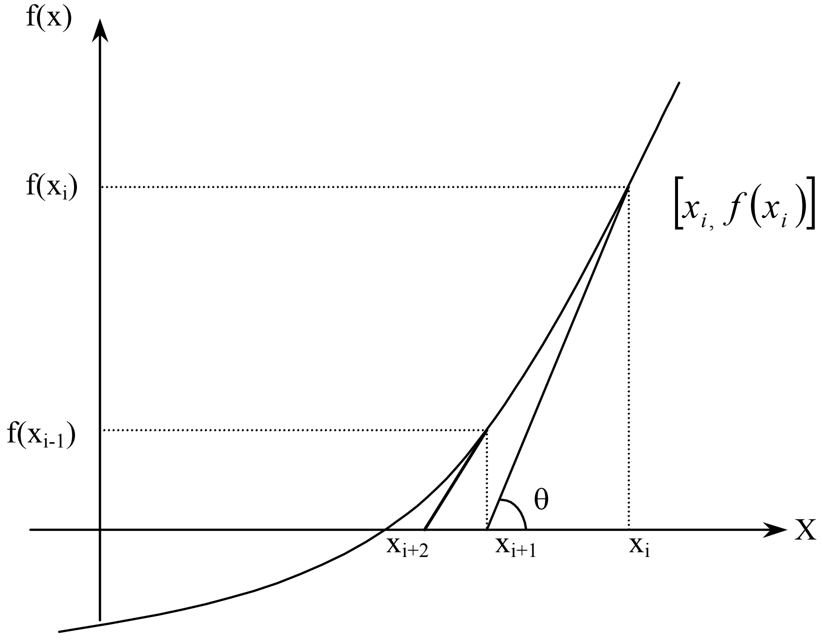
Roots of a Nonlinear Equation

Topic: Secant Method

Major: Chemical Engineering

Secant Method



Newton's Method

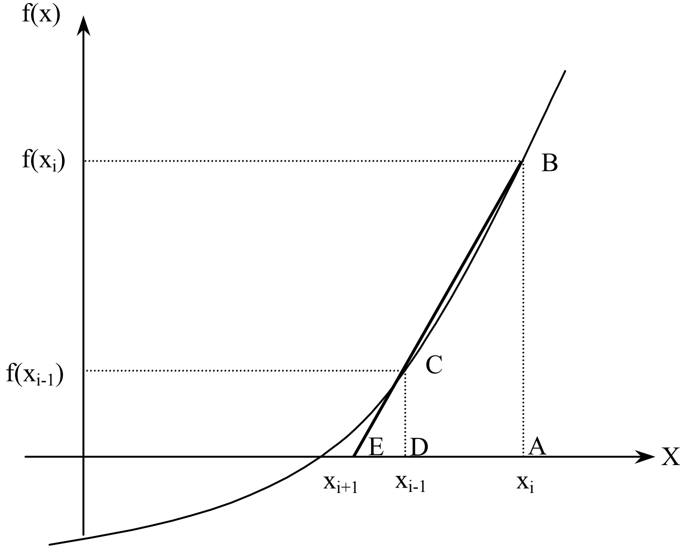
$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Approximate the derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Secant Method



Geometric Similar Triangles

$$\frac{AB}{AE} = \frac{DC}{DE}$$

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Algorithm for Secant Method

Step 1

Calculate the next estimate of the root from two initial guesses

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Find the absolute relative approximate error

$$|\epsilon_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \text{ x } 100$$

Step 2

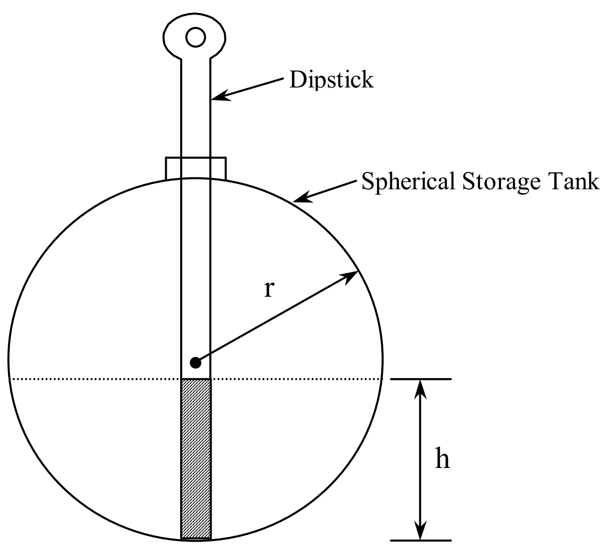
Find if the absolute relative approximate error is greater than the prespecified relative error tolerance.

If so, go back to step 1, else stop the algorithm.

Also check if the number of iterations has exceeded the maximum number of iterations.

Example

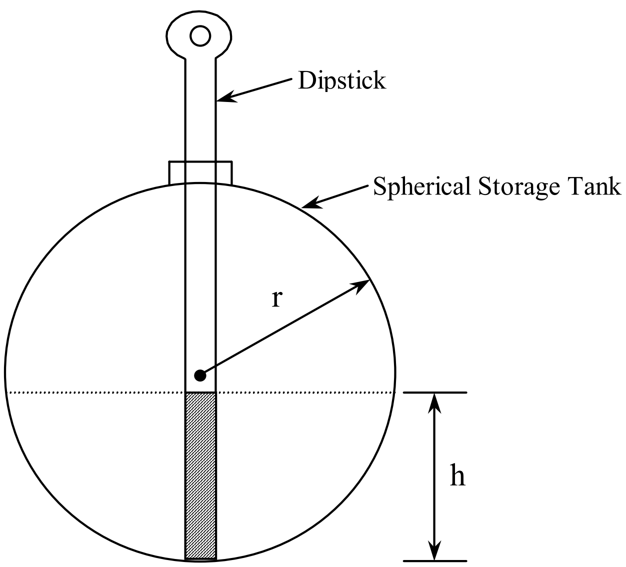
- You have a spherical storage tank containing oil. The tank has a diameter of 6 ft. You are asked to calculate the height 'h' to which a dipstick 8 ft long would be wet with oil when immersed in the tank when it contains 4 ft^3 of oil.



Solution

The equation that gives the height 'h' of liquid in the spherical tank for the given volume and radius is given by:

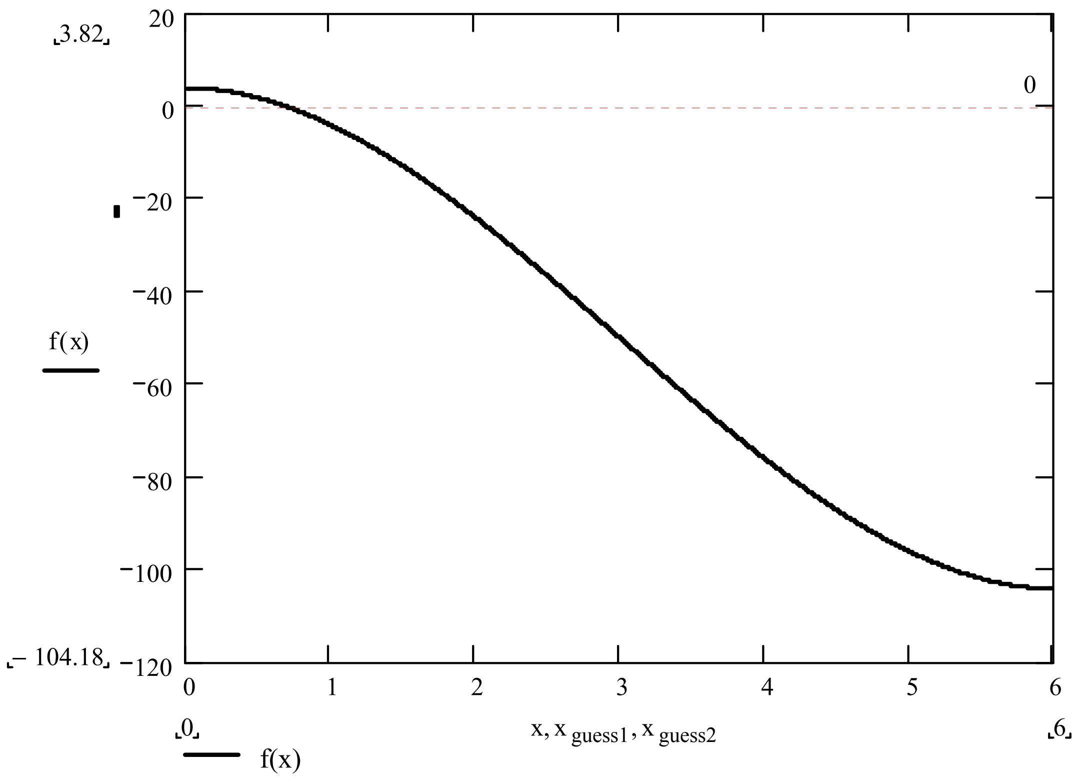
$$f(h) = h^3 - 9h^2 + 3.8197 = 0$$



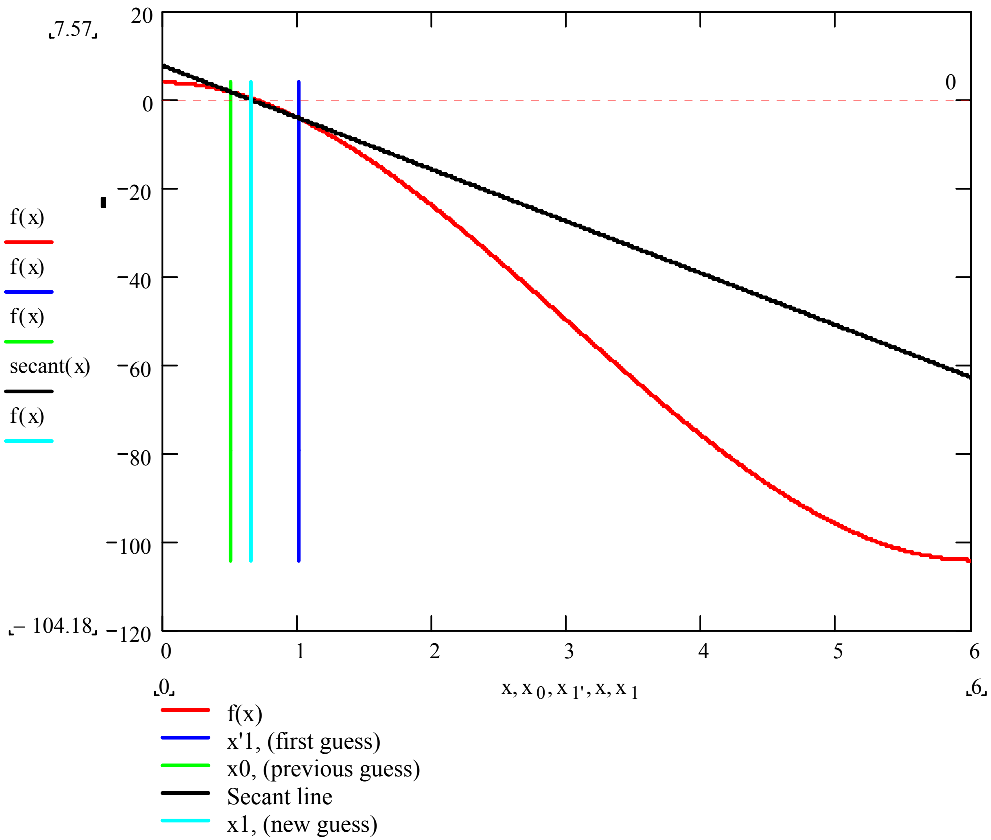
Use the secant method of finding roots of equations to find the depth 'h' to which the ball is submerged under water. Conduct three iterations to estimate the root of the above equation.

Graph of function f(x)

$$f(h) = h^3 - 9h^2 + 3.8197 = 0$$



Iteration #1



$$h_{-1} = 0.5, h_0 = 1$$

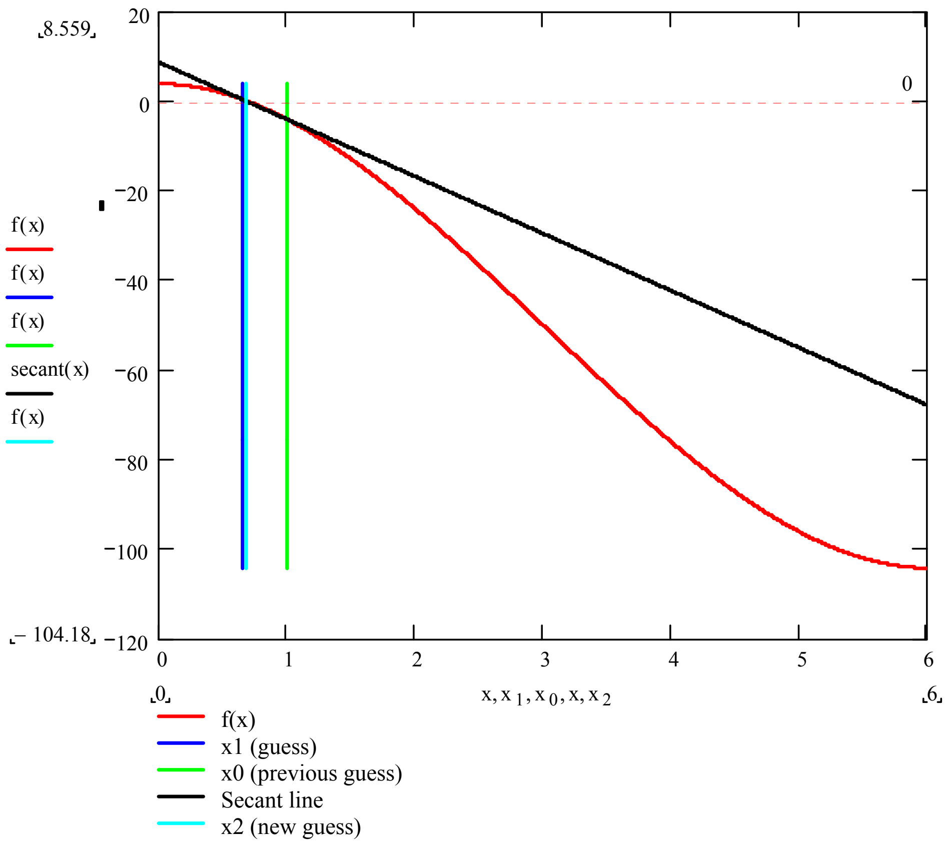
$$h_1 = h_0 - \frac{f(h_0)(h_0 - h_{-1})}{f(\mathrm{h}_0) - f(h_{-1})}$$

$$h_1 = 1 - \frac{(-4.1803)(1-0.5)}{(-4.1803)-(1.6947)}$$

$$= 0.6442$$

$$|\in_a| = 55.23\%$$

Iteration #2



$$h_0 = 1, h_1 = 0.6442$$

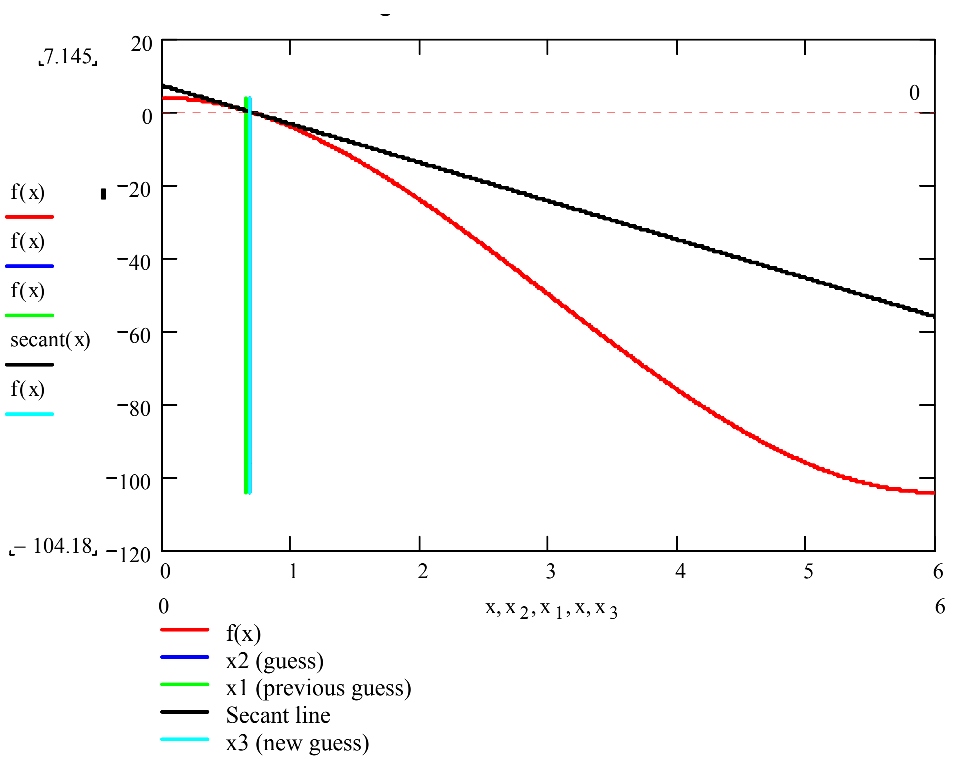
$$h_2 = h_1 - \frac{f(h_1)(h_1 - h_0)}{f(\mathrm{h}_1) - f(h_0)}$$

$$h_2 = 0.6442 - \frac{0.35179(0.6442 - 1)}{(0.35179) - (-4.1803)}$$

$$= 0.6718$$

$$|\in_a| = 4.108\%$$

Iteration #3



$$h_1 = 0.6442, h_2 = 0.6718$$

$$h_3 = h_2 - \frac{f(h_2)(h_2 - h_1)}{f(\mathrm{h}_2) - f(h_1)}$$

$$h_3 = 0.6718 - \frac{(0.06057)(0.6718 - 0.6442)}{(0.06057) - (0.35179)}$$

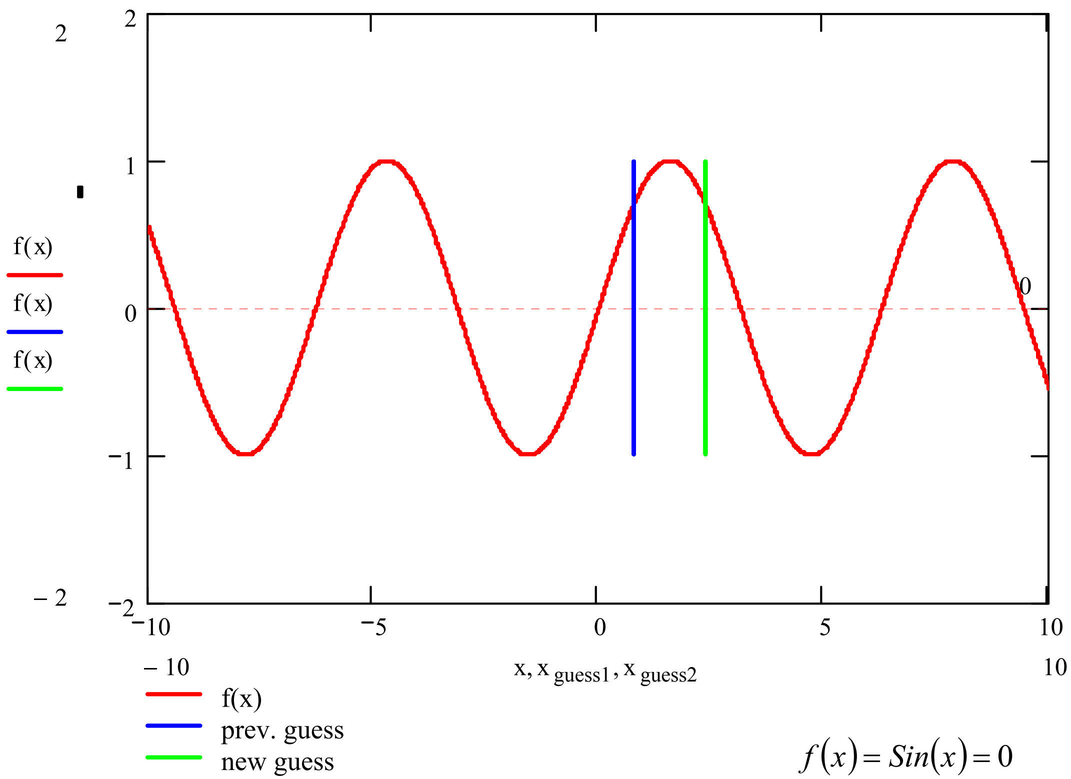
$$= 0.6775$$

$$|\in_a| = 0.841\%$$

Advantages

- Converges fast, if it converges
- Requires two guesses that do not need to bracket the root

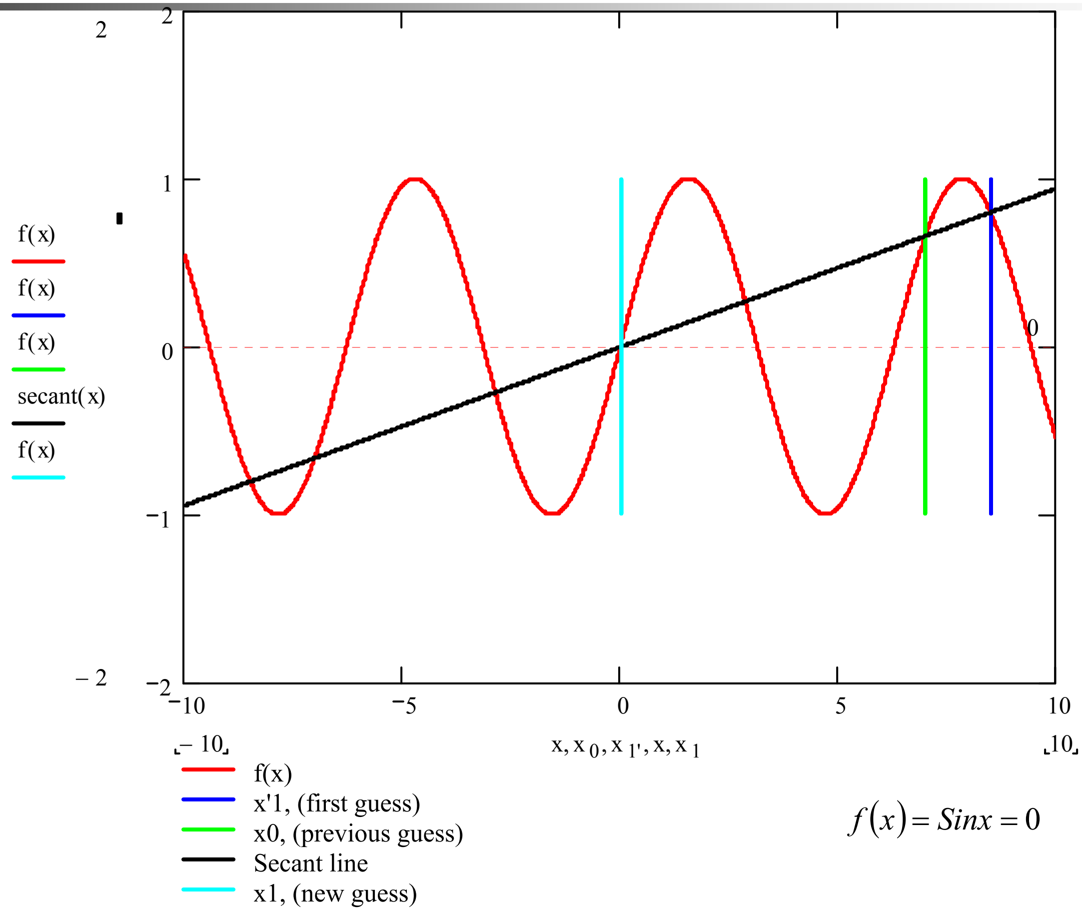
Drawbacks



$f(x) = Sin(x) = 0$

Division by zero

Drawbacks (continued)



$$f(x) = Sinx = 0$$

Root Jumping