Chapter 05.00B

Physical Problem of Interpolation Chemical Engineering

Problem Statement

Well, I am from India and we are in the habit of drinking "afternoon tea." The other day, my wife asked me to heat up some water in our kettle. I put 4 cups (you cannot have just 1 cup) of water in the kettle and put it on over our new flat-top burner.

"You are an engineer", quipped my wife teasingly. "Can you estimate how long it would take for the water to boil and the kettle to make that whistling sound?" Yes, she clearly knows that whistling sound reminds me of all the horror movies that keep me awake at night.

Figure 1. A kettle of water on a flat burner.

Solution

A cup of water is about 200 ml in volume. So the total volume of water is about 800 ml. The burner for our flat top is rated at 1200 W. From first law of thermodynamics [1],

$$\Delta H + \Delta E_p + \Delta E_k = Q - W_{sh}$$

where

ΔH = change in internal energy,

ΔE_p = change in potential energy,

ΔE_k =change in kinetic energy,

Q =heat added to the system,

W_{sh} =work that is called shaft work.

In this example

ΔE_p =0,

ΔE_k =0,

W_{sh} =0,

giving

$$\Delta H = Q.$$

Assuming no heat is lost as the kettle is assumed to be thermally insulated, the amount of heat needed is

$$Q = \Delta H$$
$$= mC_p \Delta T$$

where

m = mass of water (kg),

C_p = specific heat $\left(\frac{\text{J}}{\text{kg} - {}^\circ \text{C}} \right)$,

ΔT = change in temperature,

and the values are given as

$$m = 800\text{ml} \times \frac{1\text{kg}}{1\text{lit}}$$
$$= 0.8kg$$

$C_p = 4814 \frac{\text{T}}{\text{Kg} - {}^\circ \text{K}}$ from Table 8.1 of [1]

$$\Delta T = 100^\circ \text{C} - 22^\circ \text{C}$$
$$= 78^0 \text{C}$$

Assuming that the room temperature is 22^0C and the boiling temperature of water is 100^0C.

$$Q = mC_p \Delta T$$
$$= (0.8)(4814)(78)$$
$$= 300394 \text{J}.$$

Since the wattage of the heater is 1200 W, the time it would take to boil is

$$= \frac{300394}{1200}$$
$$\cong 250 \text{ s}$$
$$\cong 4 \text{min} 10 \text{s}$$

But, I do not see any interpolation here. One of the approximations made in the above formula is that the specific heat is constant over the temperature range of 22^0 C to 100^0 C. But it is not a constant given in the Table 1.

Table 1. Specific heat of water as a function of temperature [2].

Temperature	Specific heat
^0C	$\dfrac{\text{J}}{\text{kg.}^\circ\text{C}}$
22	4181
42	4179
52	4186
82	4199
100	4217

One assumption one may make is to use the specific heat at the average temperature. In this case it is $\dfrac{22+100}{2} = 61^0\text{ C}$.

So how do we find $C_p(61^\circ\text{ C})$? We use interpolation to do that, that is, finding the value of a discrete function at a point that is not given to us. Using $C_p(61^\circ\text{ C})$ will give us a better estimate of how much time it would take to boil the water.

References

1. Levenspiel, Octave, Understanding Engineering Thermo, Prentice Hall, New Jersey, 1996.
2. Incropera, F.P. and DeWitt, D.P., Introduction to Heat Transfer, Wiley, 4th edition, 2001.

QUESTIONS

1. Using the specific heat at the average temperature, how much is the difference in the estimated time for boiling the water.
2. Use first, second and third order polynomial interpolation to estimate $C_p(61^\circ\text{ C})$ by all the methods (except spline) you learned in class. What is the absolute relative approximate error for each order of polynomial approximation? How many significant digits are at least correct in your solution.
3. Just by looking at the data in Table 1, it may be clear that the calculated time using interpolation will not be very different from that found using the approximate specific heat. But in case of solids, it can be quite a different story. For example, to calculate heat required to raise the temperature of graphite from room temperature to 800°C for pyrolization, one needs to use proper specific heat data. Check yourself to see the difference between using specific heat at room temperature and specific heat at average temperature for the following problem. Find the heat required to raise the

temperature of 1 kg of graphite from room temperature of 22°C to 800°C, given the table of specific heat vs. temperature below.

Table 2 Specific heat of graphite as a function of temperature.

Temperature ^{0}C	Specific heat $\frac{\text{J}}{\text{kg.}^\circ\text{C}}$
-73	420
127	1070
327	1370
527	1620
727	1820

INTERPOLATION	
Topic	Physical Problem
Summary	Textbook notes of a real world problem using interpolation.
Major	Chemical Engineering
Authors	Autar Kaw
Date	December 7, 2008
Web Site	http://numericalmethods.eng.usf.edu