Holistic Numerical Methods Institute

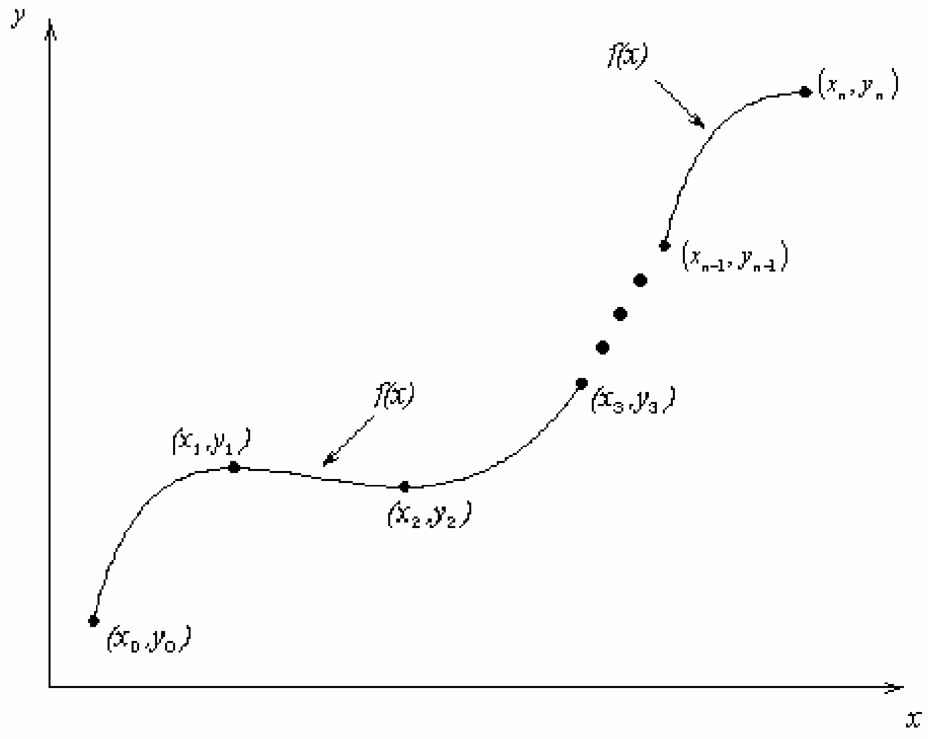
Interpolation

Topic: Direct Method

Major: Chemical

What is Interpolation ?

Given (x_0,y_0), (x_1,y_1), …… (x_n,y_n), find the value of 'y' at a value of 'x' that is not given.



Interpolants

Polynomials are the most common choice of interpolants because they are easy to:

- Evaluate
- Differentiate, and
- Integrate.

Direct Method

Given 'n+1' data points (x_0,y_0), (x_1,y_1),............. (x_n,y_n), pass a polynomial of order 'n' through the data as given below:

$$y = a_0 + a_1 x + ... + a_n x^n.$$

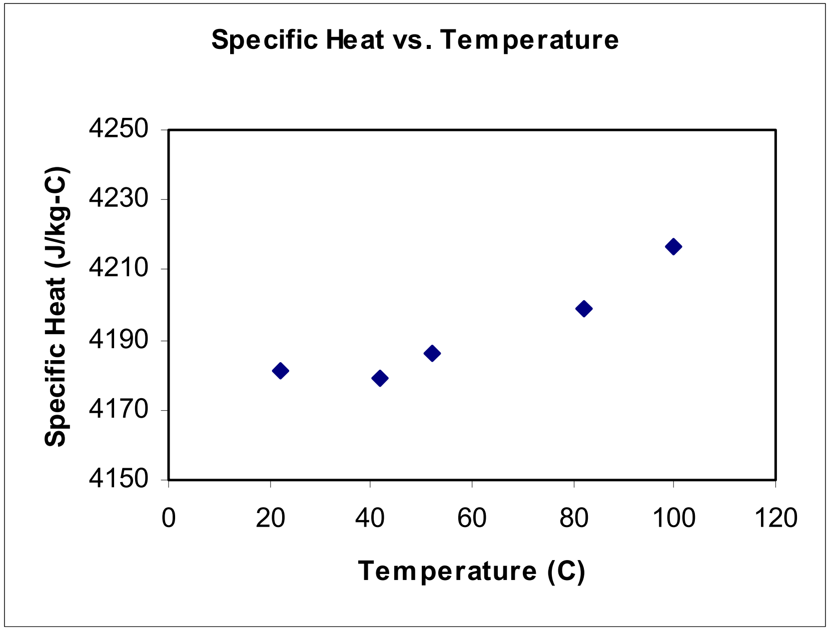
where a_0, a_1,.................. a_n are real constants.

- Set up 'n+1' equations to find 'n+1' constants.
- To find the value 'y' at a given value of 'x', simply substitute the value of 'x' in the above polynomial.

Example

To find how much heat is required to bring a kettle of water to boiling point, you are asked to calculate the specific heat of water at 61^0C. Use linear, quadratic and cubic interpolation.

Temperature	Specific heat
°C	$\frac{J}{kg - ^\circ C}$
22	4181
42	4179
52	4186
82	4199
100	4217

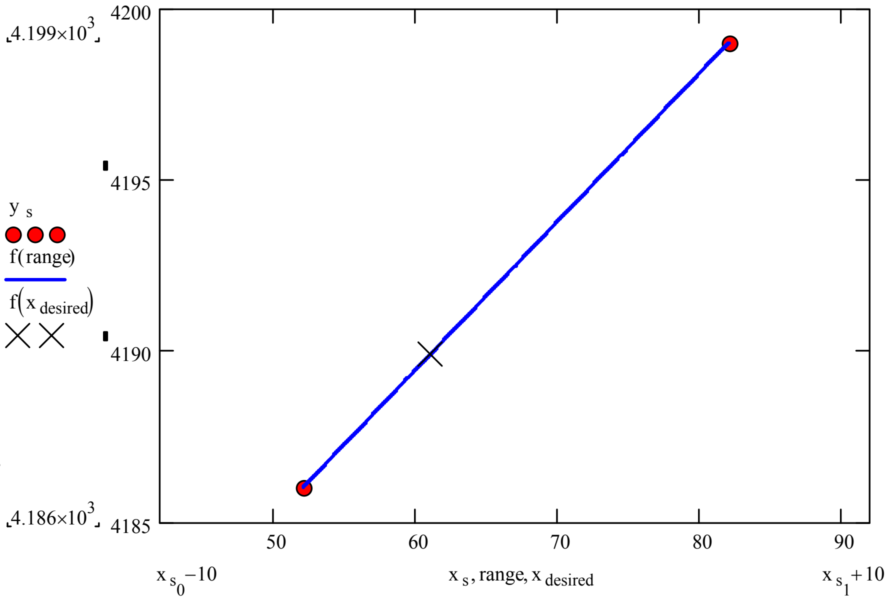


Linear Interpolation

$$C_p(T) = a_0 + a_1 T$$

$$C_p(52) = a_0 + a_1(52) = 4186$$

$$C_p(82) = a_0 + a_1(82) = 4199$$



Solving the above two equations gives,

$$a_0 = 4163.5 \qquad a_1 = 0.43333$$

Hence

$$C_p(T) = 4163.5 + 0.43333t, \quad 52 \le T \le 82.$$

$$C_p(61) = 4163.5 + 0.43333(61) = 4189.9 \frac{J}{kg - ^\circ C}$$

Quadratic Interpolation

$$C_p(T) = a_0 + a_1 T + a_2 T^2$$

$$C_p(42) = a_0 + a_1(42) + a_2(42)^2 = 4179$$

$$C_p(52) = a_0 + a_1(52) + a_2(52)^2 = 4186$$

$$C_p(82) = a_0 + a_1(82) + a_2(82)^2 = 4199$$

Solving the above three equations gives

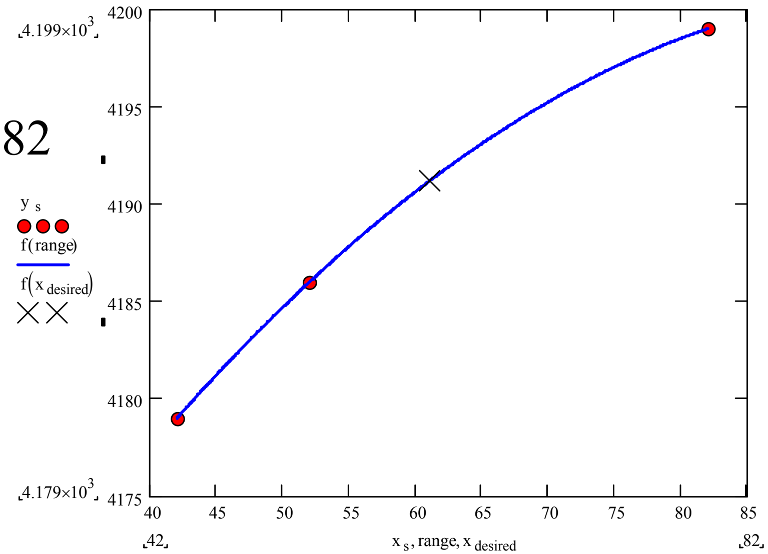
$$a_0 = 4135.0 \qquad a_1 = 1.3267 \qquad a_2 = -6.6667 * 10^{-3}$$

Quadratic Interpolation (contd)

$$C_p(T) = 4135.0 + 1.3267T - 6.6667*10^{-3}T^2,\ \ 42 \le T \le 82$$

$$C_p(61) = 4135.0 + 1.3267(61) - 6.6667*10^{-3}(61)^2$$

$$= 4191.2 \frac{J}{kg - {}^\circ C}$$



The absolute relative approximate error $|\in_a|$ obtained between the results from the first and second order polynomial is

$$|\in_a| = \left|\frac{4191.2 - 4189.9}{4191.2}\right| \times 100$$

$$= 0.031017\%$$

Cubic Interpolation

$$C_p(T) = a_0 + a_1 T + a_2 T^2 + a_3 T^3$$

$$C_p(42) = 4179 = a_0 + a_1(42) + a_2(42)^2 + a_3(42)^3$$

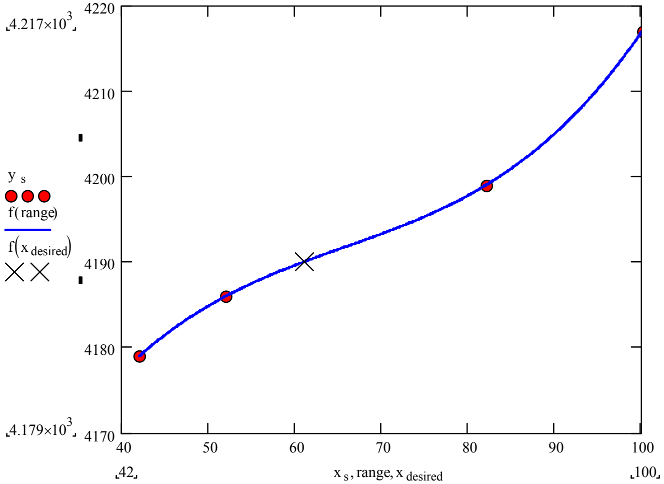
$$C_p(52) = 4186 = a_0 + a_1(52) + a_2(52)^2 + a_3(52)^3$$

$$C_p(82) = 4199 = a_0 + a_1(82) + a_2(82)^2 + a_3(82)^3$$

$$C_p(100) = 4217 = a_0 + a_1(100) + a_2(100)^2 + a_3(100)^3$$

$a_0 = 4078$ $a_1 = 4.4771$ $a_2 = -0.06272$ $a_3 = 3.1849 * 10^{-4}$

Cubic Interpolation (contd)



$$C_p(T) = 4078 + 4.4771T - 0.06272T^2 + 3.1849 * 10^{-4} T^3, \quad 42 \le T \le 100$$

$$T(61) = 4078 + 4.4771(61) - 0.06272(61)^2 + 3.1849 * 10^{-4}(61)^3 = 4190.0 \frac{J}{kg - ^\circ C}$$

The absolute relative approximate error $|\in_a|$ obtained between the results from the second and third order polynomial is

$$|\in_a| = \left|\frac{4190.0 - 4191.2}{4190.0}\right| \times 100$$

$$= 0.028640\%$$

Comparison Table

Order of Polynomial	1	2	3
$C_p(T)$ $\frac{J}{kg-^{\circ}C}$	4189.9	4191.2	4190.0
Absolute Relative Approximate Error	----------	0.031017%	0.028640%