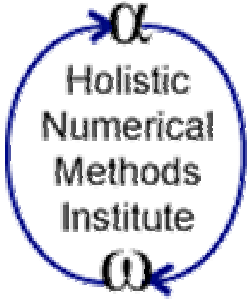


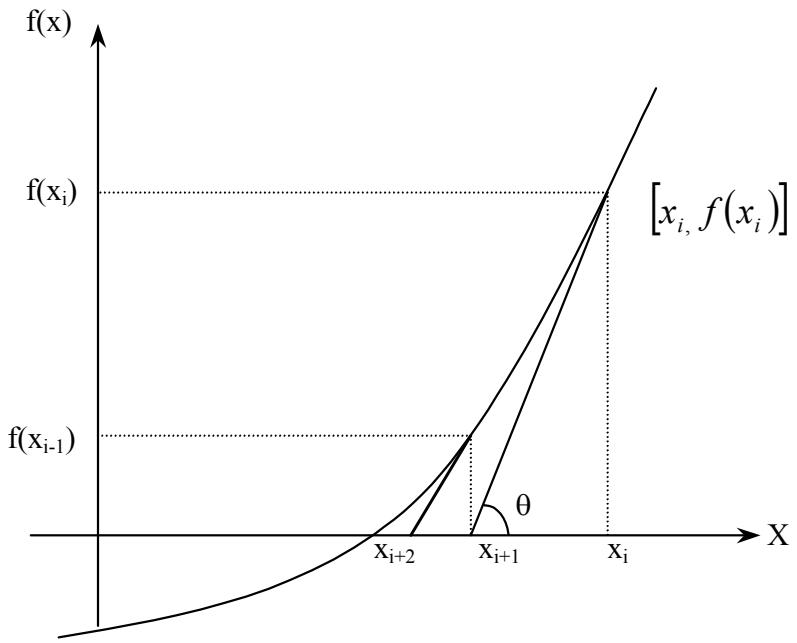
Roots of a Nonlinear Equation

Topic: Newton-Raphson Method

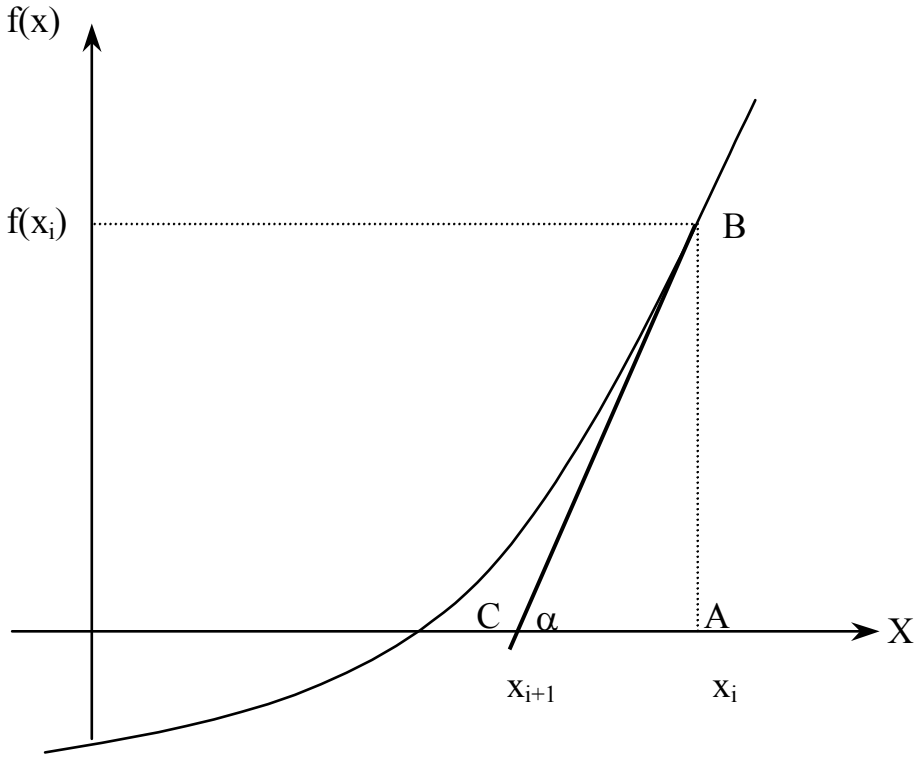
Major: Civil Engineering

Newton-Raphson Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$



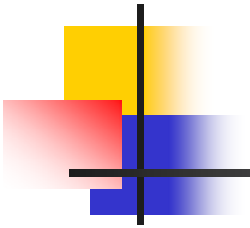
Derivation



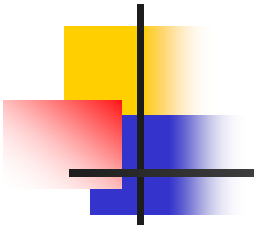
$$\tan(\alpha) = \frac{AB}{AC}$$

$$f'(x_i) = \frac{f(x_i)}{x_i - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$



Algorithm for Newton-Raphson Method



Step 1

Evaluate f′(x) symbolically

Step 2

Calculate the next estimate of the root

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Find the absolute relative approximate error

$$\left|\epsilon_a\right| = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \text{ x } 100$$

Step 3

- Find if the absolute relative approximate error is greater than the pre-specified relative error tolerance.
- If so, go back to step 2, else stop the algorithm.
- Also check if the number of iterations has exceeded the maximum number of iterations.

Example

- You are making a bookshelf to carry books that range from 8 ½ ″ to 11″ in height and would take 29″of space along length. The material is wood having Young's Modulus 3.667 Msi, thickness 3/8 ″ and width 12″. You are asked to find the maximum deflection of the bookshelf.

The vertical deflection of the shelf is given by

$$v(x) = 0.42493 \text{x} 10^{-4} x^3 - 0.13533 \text{x} 10^{-8} x^5 - 0.66722 \text{x} 10^{-6} x^4 - 0.018507x$$

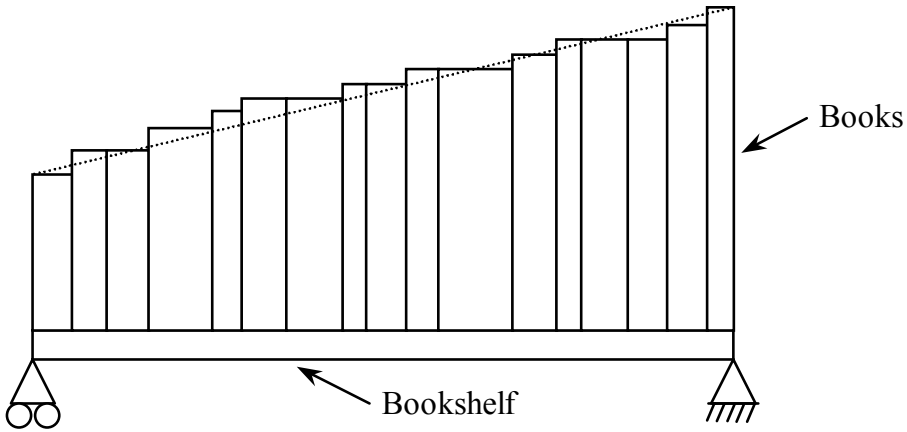
where x is the position where the deflection is maximum. Hence to find the maximum deflection we need to find where $f(x) = \frac{dv}{dx} = 0$

Solution

The equation that gives the position 'x' where the deflection is maximum is given by:

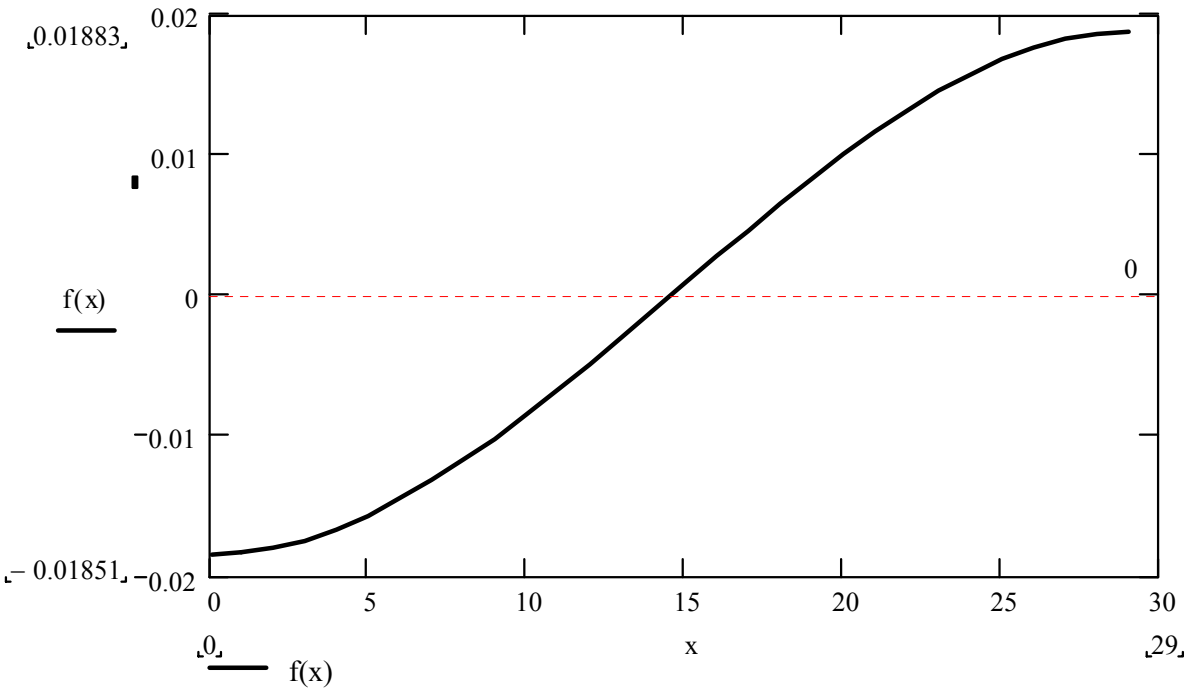
$$f(x) = -0.67665 \times 10^{-8} x^4 - 0.26689 \times 10^{-5} x^3 + 0.12748 \times 10^{-3} x^2 - 0.018507 = 0$$

Use the bisection method of finding roots of equations to find the depth 'x' to which the ball is submerged under water. Conduct three iterations to estimate the root of the above equation.

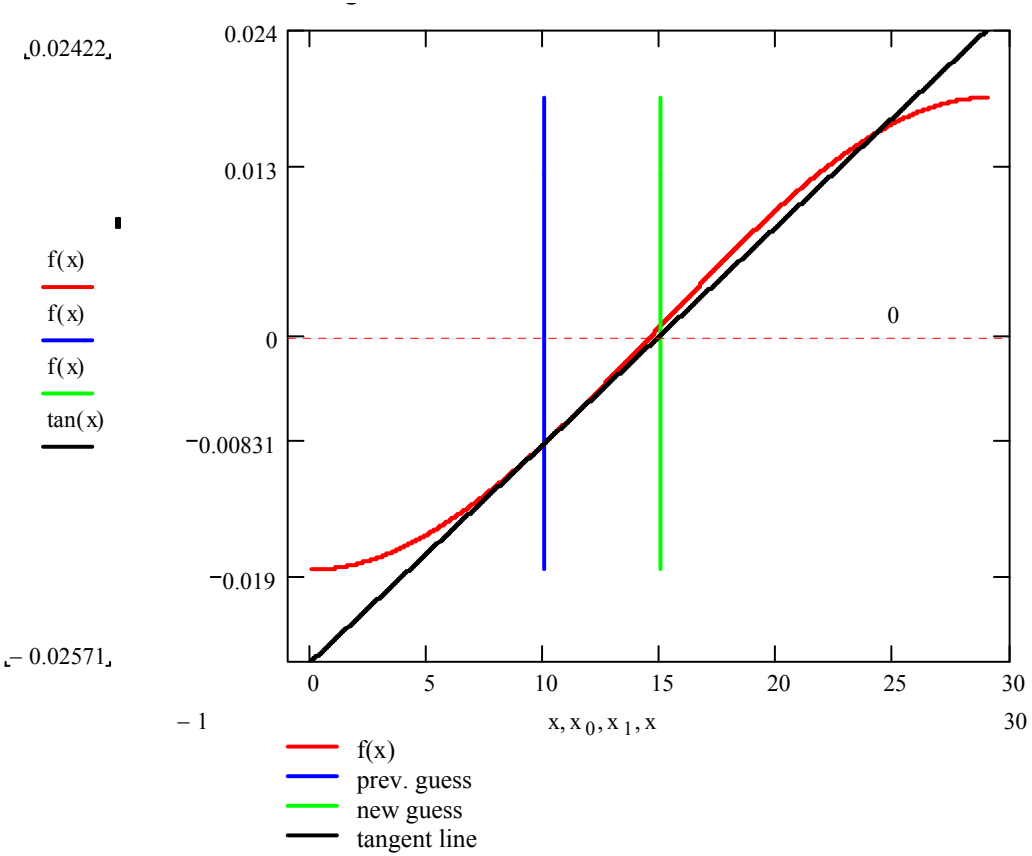


Graph of function f(x)

$$f(x) = -0.67665 \text{ x } 10^{-8} x^4 - 0.26689 \text{ x } 10^{-5} x^3 + 0.12748 \text{ x } 10^{-3} x^2 - 0.018507 = 0$$



Iteration #1



$$x_0 = 10$$

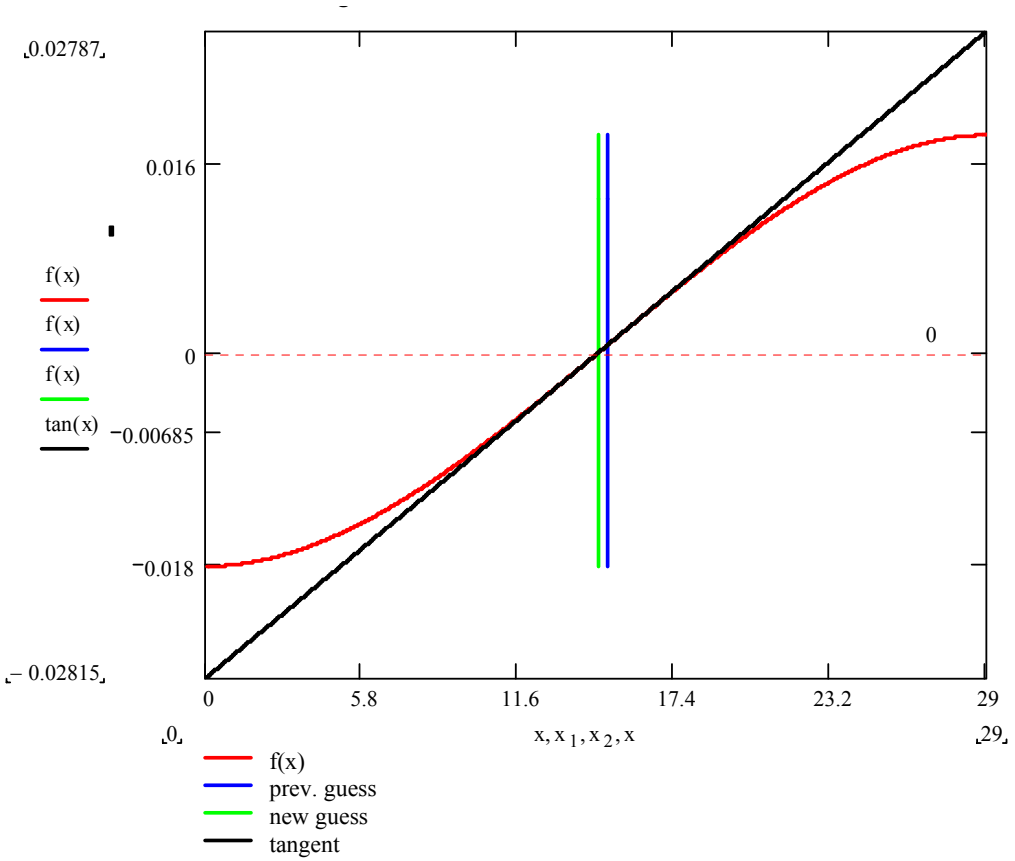
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 10 - \frac{-8.49556 \text{ x}10^{-3}}{1.721866 \text{ x}10^{-5}}$$

$$= 14.93393$$

$$|\in_a| = 33.038\ \%$$

Iteration #2



$$x_1 = 14.93393$$

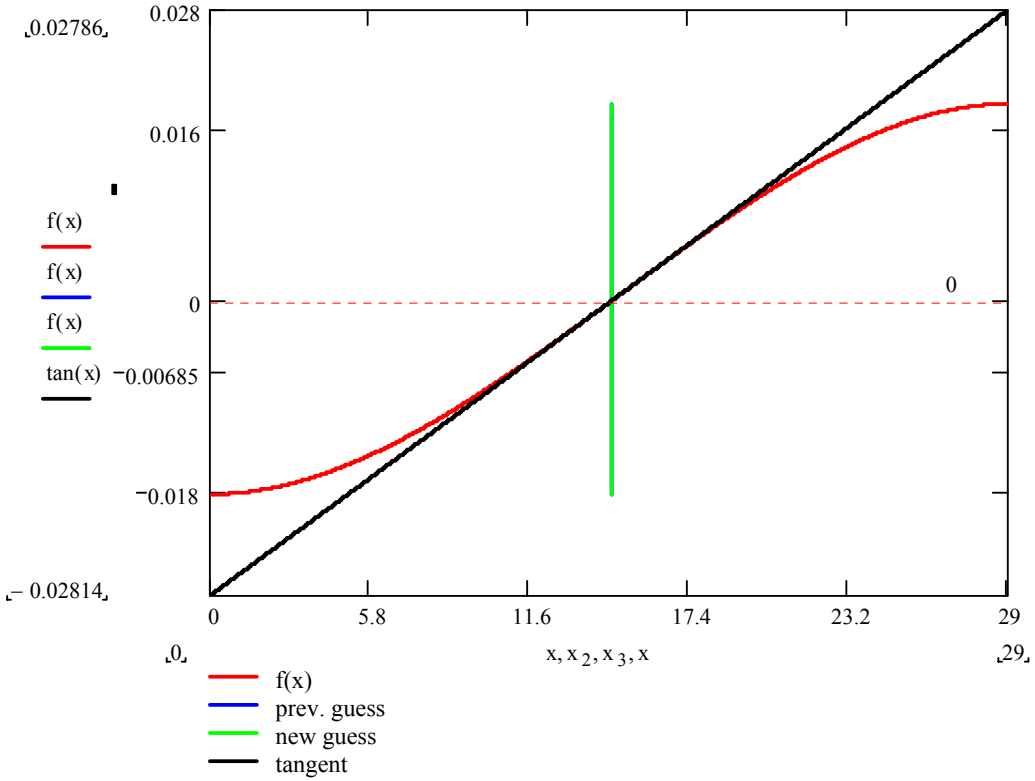
$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 14.93393 - \frac{6.98294 \text{ x}10^{-4}}{1.93174 \text{ x}10^{-3}}$$

$$= 14.57245$$

$$|\in_a| = 2.48061\ \%$$

Iteration #3



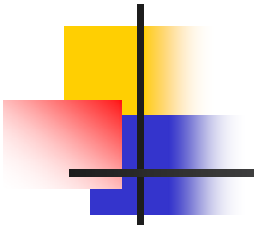
$$x_2 = 14.57245$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 14.57245 - \frac{-4.70776 \text{ x}10^{-9}}{1.93136 \text{ x}10^{-4}}$$

$$= 14.57242$$

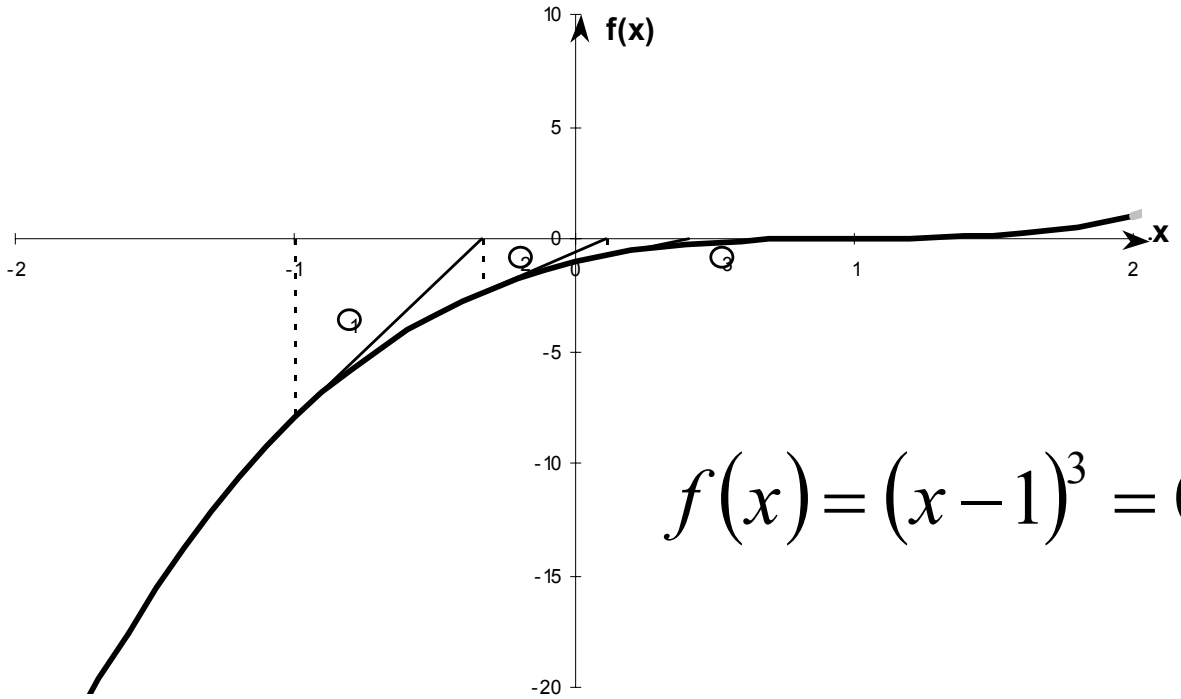
$$|\epsilon_a| = 1.6727 \text{ x}10^{-5}\%$$



Advantages

- Converges fast, if it converges
- Requires only one guess

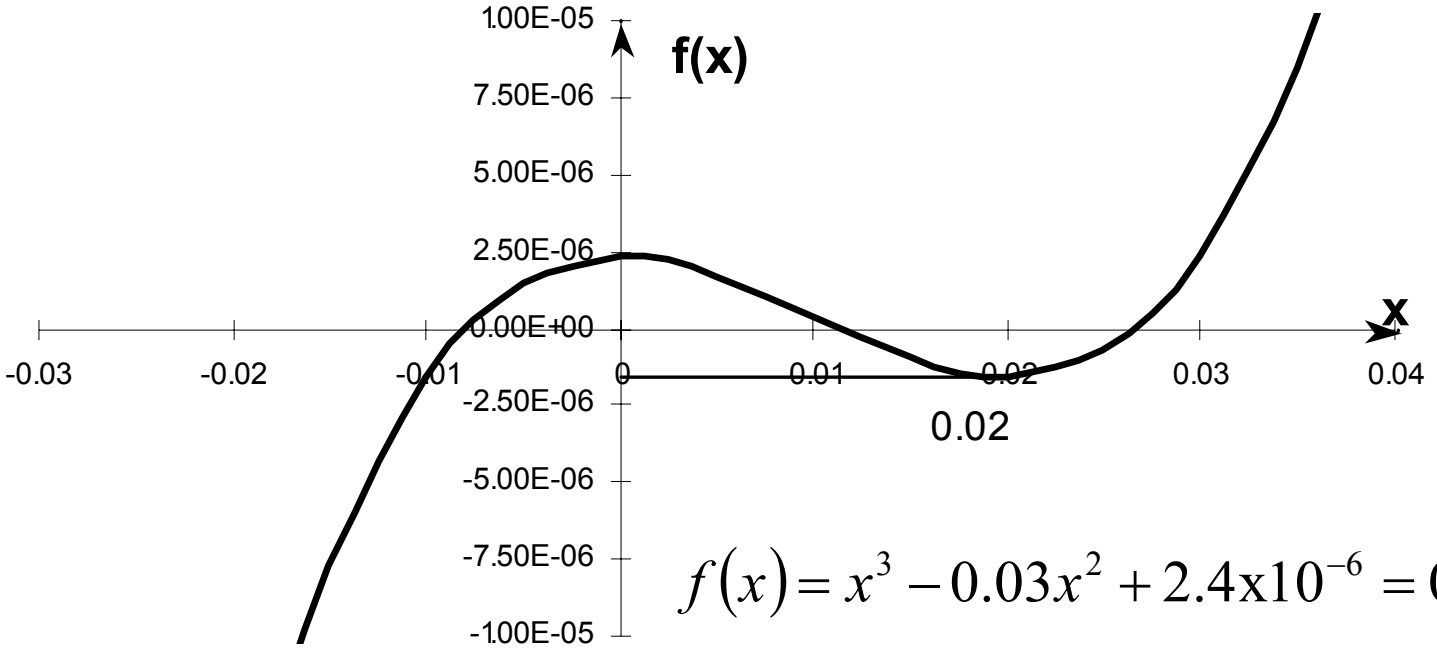
Drawbacks



$$f(x) = (x-1)^3 = 0$$

Inflection Point

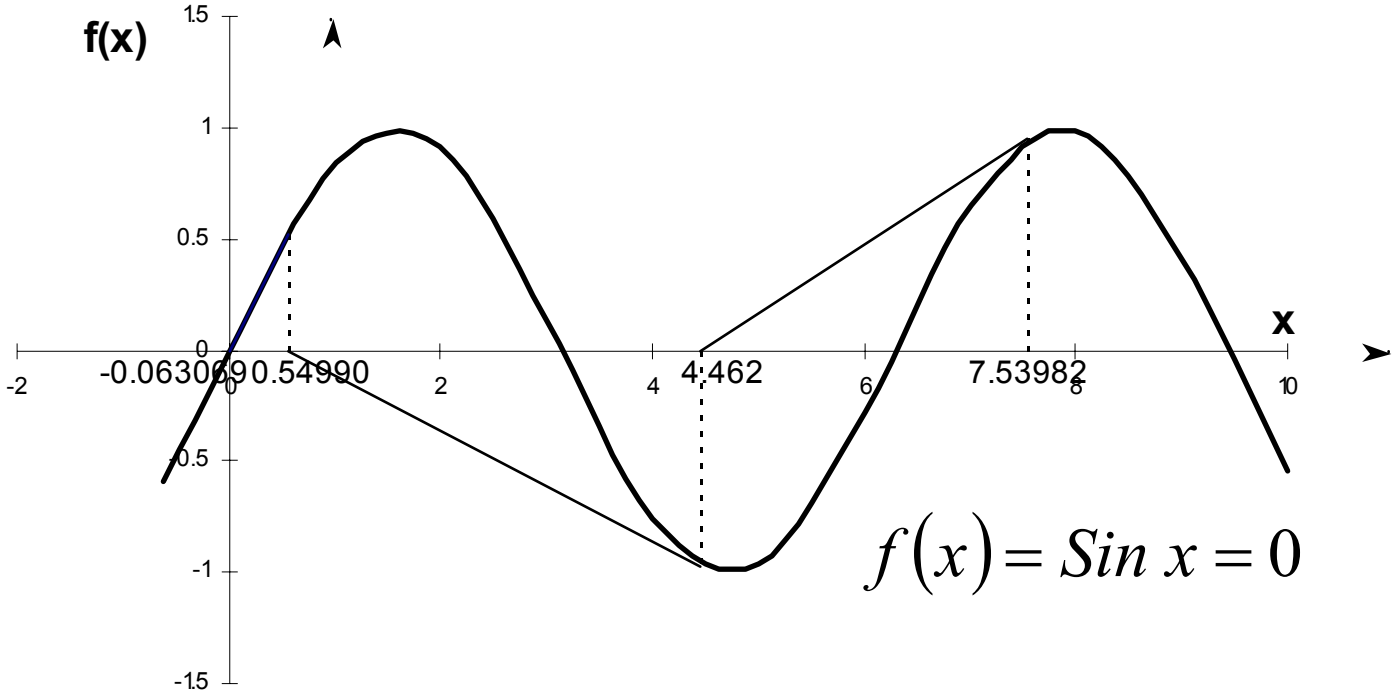
Drawbacks (continued)



$$f(x) = x^3 - 0.03x^2 + 2.4\text{x}10^{-6} = 0$$

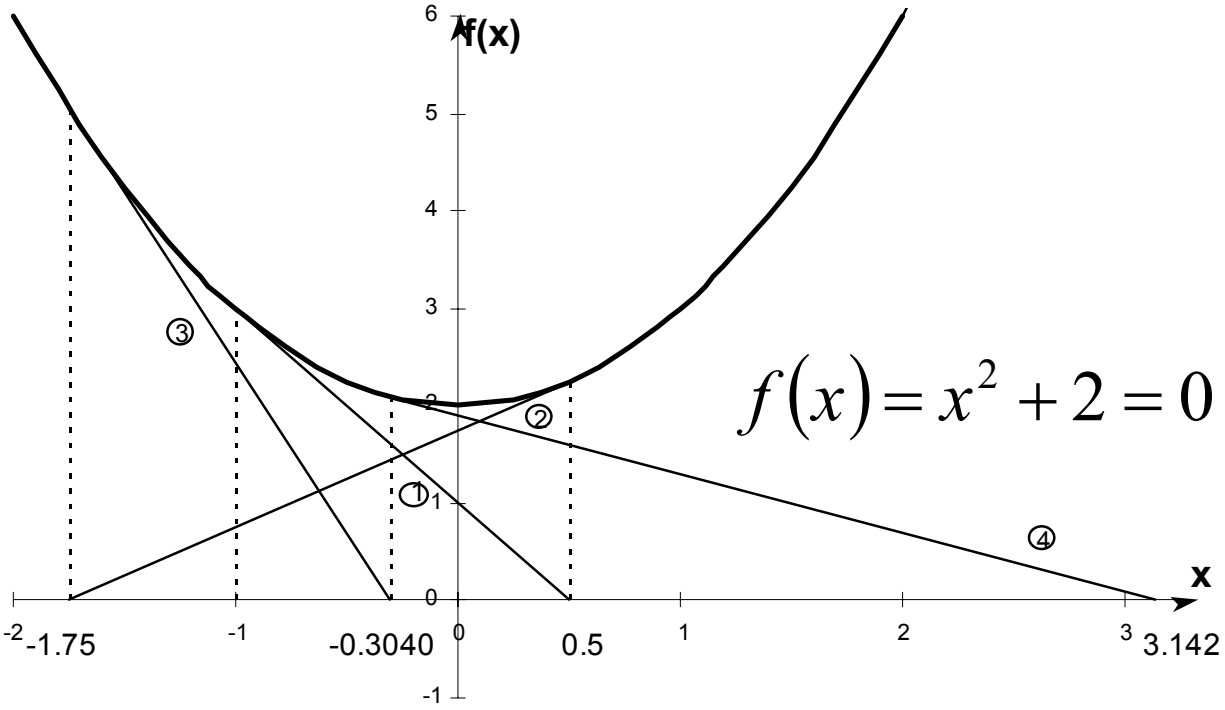
Division by zero

Drawbacks (continued)



Root Jumping

Drawbacks (continued)



Oscillations near Local Maxima or Minima