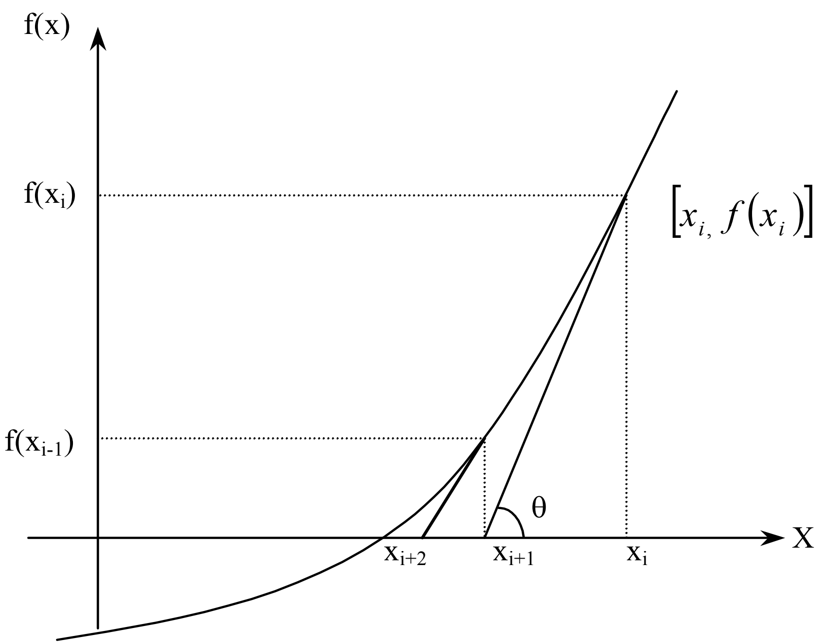
Roots of a Nonlinear Equation

Topic: Secant Method

Major: Civil Engineering

Secant Method



Newton's Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

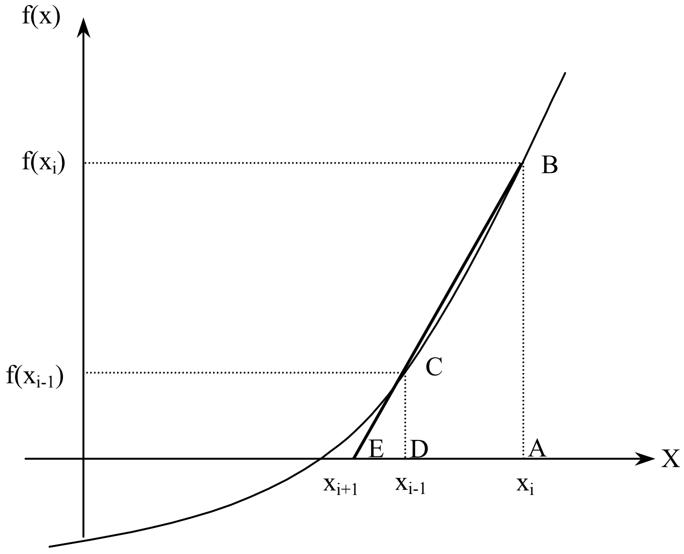
Approximate the derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Secant Method

Geometric Similar Triangles



$$\frac{AB}{AE} = \frac{DC}{DE}$$

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Algorithm for Secant Method

Step 1

Calculate the next estimate of the root from two initial guesses

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Find the absolute relative approximate error

$$\left|\in_a\right| = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \text{ x } 100$$

Step 2

Find if the absolute relative approximate error is greater than the prespecified relative error tolerance.

If so, go back to step 1, else stop the algorithm.

Also check if the number of iterations has exceeded the maximum number of iterations.

Example

- You are making a bookshelf to carry books that range from 8 ½ ″ to 11″ in height and would take 29″of space along length. The material is wood having Young's Modulus 3.667 Msi, thickness 3/8 ″ and width 12″. You are asked to find the maximum deflection of the bookshelf.

The vertical deflection of the shelf is given by

$$v(x) = 0.42493 \text{x} 10^{-4} x^3 - 0.13533 \text{x} 10^{-8} x^5 - 0.66722 \text{x} 10^{-6} x^4 - 0.018507x$$

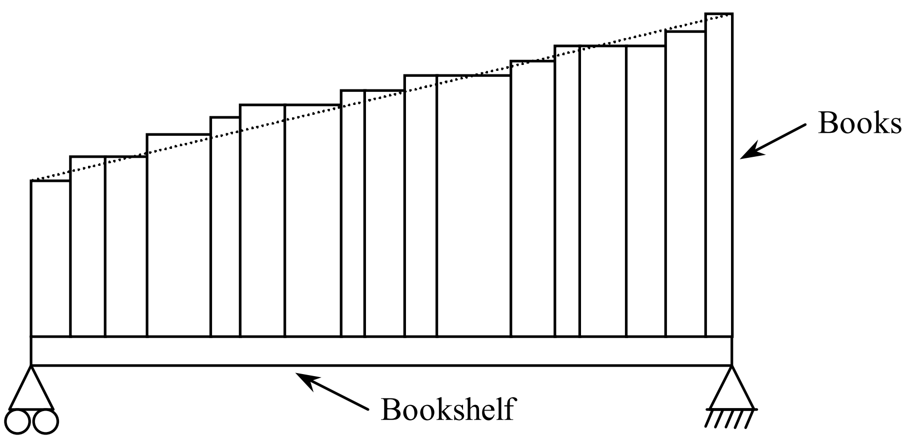
where *x* is the position where the deflection is maximum. Hence to find the maximum deflection we need to find where $f(x) = \dfrac{dv}{dx} = 0$

Solution

The equation that gives the position 'x' where the deflection is maximum is given by:

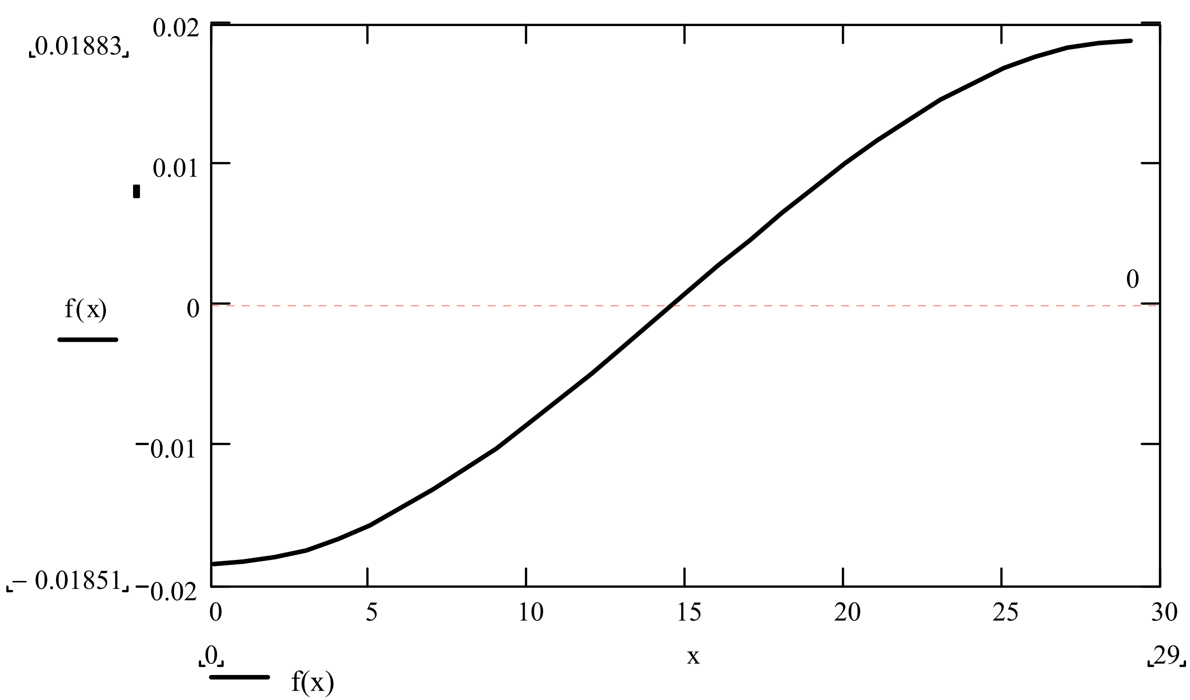
$$f(x) = -0.67665 \text{x} 10^{-8} x^4 - 0.26689 \text{x} 10^{-5} x^3 + 0.12748 \text{x} 10^{-3} x^2 - 0.018507 = 0$$

Use the Secant method of finding roots of equations to find the position 'x' where the beam deflects the most. Conduct three iterations to estimate the root of the above equation.

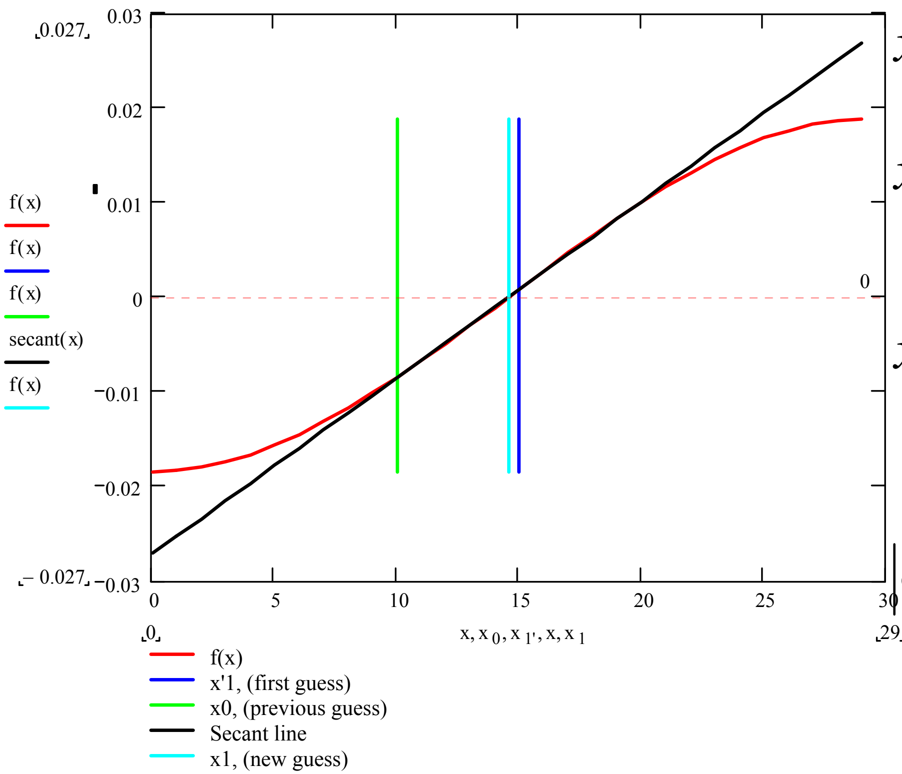


Graph of function f(x)

$$f(x) = -0.67665 \times 10^{-8} x^4 - 0.26689 \times 10^{-5} x^3 + 0.12748 \times 10^{-3} x^2 - 0.018507 = 0$$



Iteration #1



$$x_{-1} = 10, x_0 = 15$$

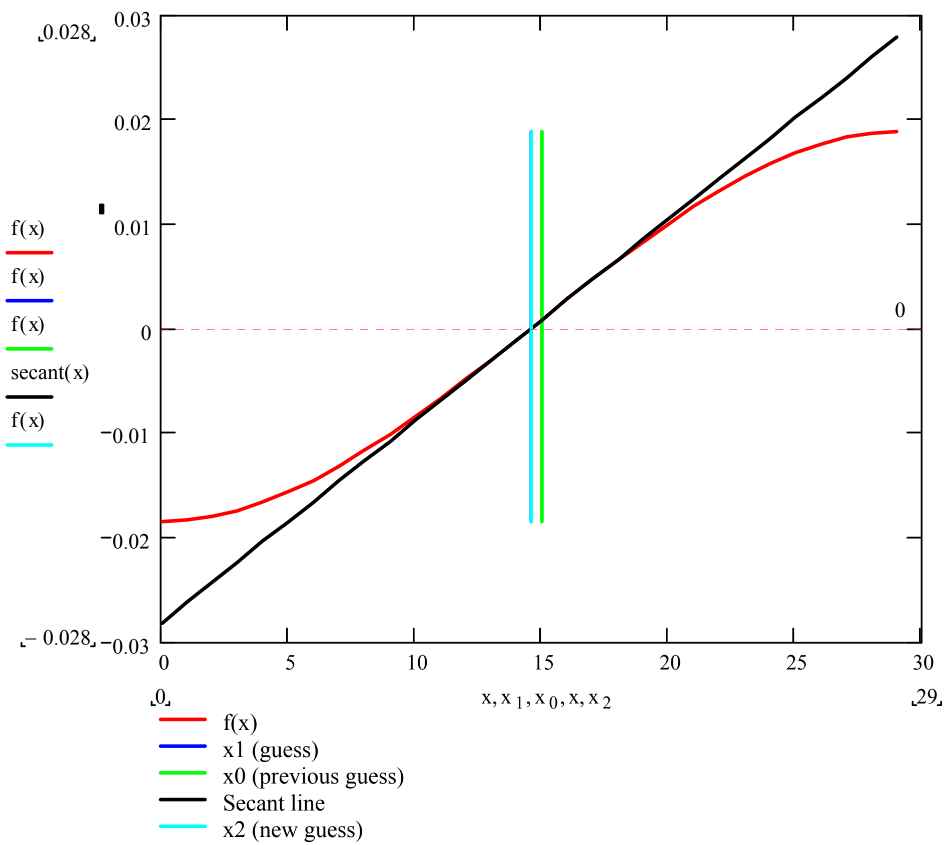
$$x_1 = x_0 - \frac{f(x_0)(x_0 - x_{-1})}{f(x_0) - f(x_{-1})}$$

$$x_1 = 15 - \frac{(8.25908\text{x}10^{-4})(15-10)}{(8.25908\text{x}10^{-4}) - (-8.49556\text{x}10^{3})}$$

$$= 14.5569$$

$$|\in_a| = 3.0439\%$$

Iteration #2



$$x_0 = 15, x_1 = 14.5569$$

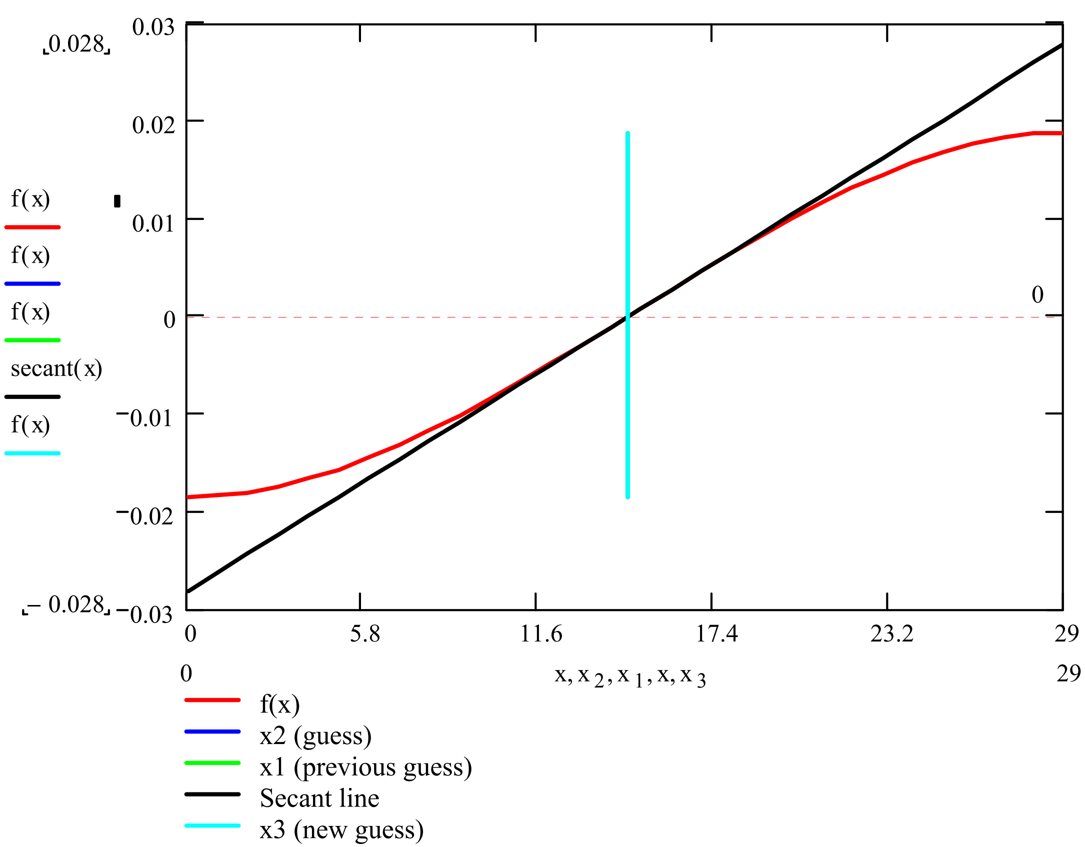
$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = 14.5569 - \frac{-2.98701\text{x}10^{-4}(14.5569 - 15)}{(-2.98701\text{x}10^{-4}) - (8.25908\text{x}10^{-4})}$$

$$= 14.57245$$

$$|\in_a| = 0.10611\%$$

Iteration #3



$x_1 = 14.5569\ ,\ x_2 = 14.57245$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$x_3 = 14.57245 - \frac{(-6.06756\text{x}10^{-9})(14.57245 - 14.5669)}{(-6.06756\text{x}10^{-9}) - (-2.98701\text{x}10^{-4})}$$

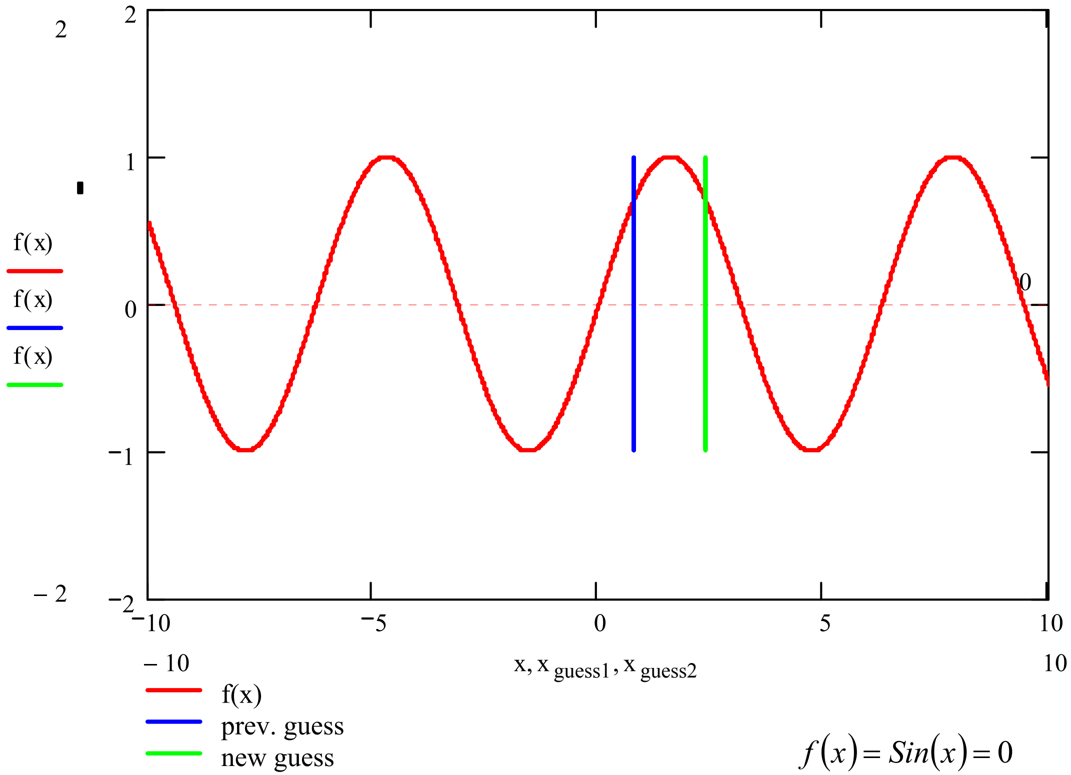
$= 14.57245$

$|\in_a| = 2.1559\ \text{x}10^{-5}\%$

Advantages

- Converges fast, if it converges
- Requires two guesses that do not need to bracket the root

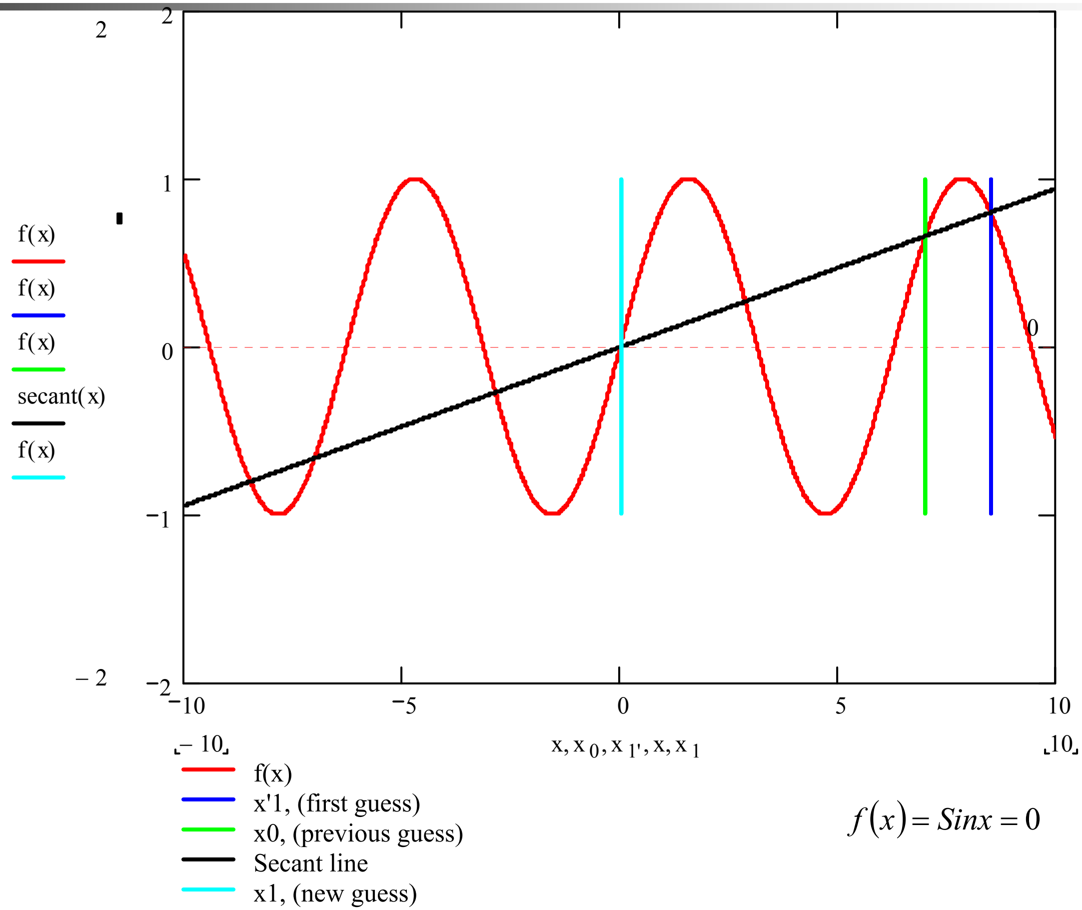
Drawbacks



$f(x) = Sin(x) = 0$

Division by zero

Drawbacks (continued)



$$f(x) = Sin x = 0$$

Root Jumping