Simultaneous Linear Equations

Holistic Numerical Methods Institute

Topic: LU Decomposition

Major: Civil Engineering

LU Decomposition

LU Decomposition is another method to solve a set of simultaneous linear equations

Which is better, Gauss Elimination or LU Decomposition?

To answer this, a closer look at LU decomposition is needed.

LU Decomposition

Method

For most non-singular matrix $[A]$ that one could conduct Naïve Gauss Elimination forward elimination steps, one can always write it as

$$[A] = [L][U]$$

Where

$[L]$ = lower triangular martix

$[U]$ = upper triangular martix

LU Decomposition

Proof

If solving a set of linear equations $[A][X]=[C]$

If $[A]=[L][U]$ Then $[L][U][X]=[C]$

Multiply by $[L]^{-1}$

Which gives $[L]^{-1}[L][U][X]=[L]^{-1}[C]$

Remember $[L]^{-1}[L]=[I]$ which leads to $[I][U][X]=[L]^{-1}[C]$

Now, if $[I][U]=[U]$ then $[U][X]=[L]^{-1}[C]$

Now, let $[L]^{-1}[C]=[Z]$

Which ends with $[L][Z]=[C]$ (1)

and $[U][X]=[Z]$ (2)

LU Decomposition

How can this be used?

Given $[A][X]=[C]$

Decompose $[\mathrm{A}]$ into $[L]$ and $[U]$

Then solve $[L][Z]=[\mathrm{C}]$ for $[Z]$

And then solve $[U][\mathrm{X}]=[Z]$ for $[X]$

LU Decomposition

How is this better or faster than Gauss Elimination?

Let's look at computational time.

n = number of equations

To decompose [A], time is proportional to $\frac{n^3}{3}$

To solve $[U][X]=[C]$ and $[\mathrm{L}][\mathrm{Z}]=[\mathrm{C}]$

time proportional to $\frac{n^2}{2}$

LU Decomposition

Therefore, total computational time for LU Decomposition is proportional to

$$\frac{n^3}{3} + 2\left(\frac{n^2}{2}\right) \quad \text{or} \quad \frac{n^3}{3} + n^2$$

Gauss Elimination computation time is proportional to

$$\frac{n^3}{3} + \frac{n^2}{2}$$

How is this better?

LU Decomposition

What about a situation where the [C] vector changes?

In LU Decomposition, LU decomposition of [A] is independent of the [C] vector, therefore it only needs to be done once.

Let m = the number of times the [C] vector changes

The computational times are proportional to

LU decomposition = $m(\frac{n^3}{3}+\frac{n^2}{2})$ Gauss Elimination= $\frac{n^3}{3}+m(n^2)$

Consider a 100 equation set with 50 right hand side vectors

LU Decomposition = 8.33×10^5 Gauss Elimination = 1.69×10^7

LU Decomposition

Another Advantage

Finding the Inverse of a Matrix

LU Decomposition

$$\frac{n^3}{3} + n(n^2) = \frac{4n^3}{3}$$

Gauss Elimination

$$n\left(\frac{n^3}{3} + \frac{n^2}{2}\right) = \frac{n^4}{3} + \frac{n^3}{2}$$

For large values of n

$$\frac{n^4}{3} + \frac{n^3}{2} \rangle\rangle \frac{4n^3}{3}$$

LU Decomposition

Method: [A] Decompose to [L] and [U]

$$[A]=[L][U]=\begin{bmatrix}1 & 0 & 0\\ \ell_{21} & 1 & 0\\ \ell_{31} & \ell_{32} & 1\end{bmatrix}\begin{bmatrix}u_{11} & u_{12} & u_{13}\\ 0 & u_{22} & u_{23}\\ 0 & 0 & u_{33}\end{bmatrix}$$

[U] is the same as the coefficient matrix at the end of the forward elimination step.

[L] is obtained using the *multipliers* that were used in the forward elimination process

Example: Cylinder Stresses

To find the maximum stresses in a compounded cylinder, the following four simultaneous linear equations need to solved.

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 4.2857\times10^{7} & -5.4619\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ -6.5 & -0.15384 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857\times10^{7} & -3.6057\times10^{5} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -7.887\times10^{3} \\ 0 \\ 0.007 \\ 0 \end{bmatrix}$$

Example: Cylinder Stresses

If in the compounded cylinder, the inner cylinder has an internal radius of *a=5"*, and outer radius *c=6.5"*, while the outer cylinder has an internal radius of *c=6.5"* and outer radius, *b=8"*. Given E=30.0x10^6 psi, v=0.3, and that the hoop stress in outer cylinder is given by:

$$\sigma_\theta^2 = \frac{E}{1-\nu^2}\left[c_3(1+\nu) + c_4\left(\frac{1-\nu}{r^2}\right)\right]$$

find the stress on the inside radius of the outer cylinder

Find the values of c_1, c_2, c_3 and c_4 using Naïve Guass Elimination

Example: Cylinder Stresses

Finding the [*U*] matrix

Using the Forward Elimination Procedure of Gauss Elimination:
Step 1

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 4.2857\times10^{7} & -5.4619\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ -6.5 & -0.15384 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857\times10^{7} & -3.6057\times10^{5} \end{bmatrix}$$

$$\text{Row2} - \left[\frac{\text{Row1}}{4.2857\times10^{7}}\right]\times(4.2857\times10^{7}) =$$

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 0 & 3.7688\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ -6.5 & -0.15384 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857\times10^{7} & -3.6057\times10^{5} \end{bmatrix}$$

Example: Cylinder Stresses

Finding the [*U*] matrix

Using the Forward Elimination Procedure of Gauss Elimination:
Step 1

$$\text{Row3} - \left[\frac{\text{Row1}}{4.2857\times10^{7}}\right]\times(-6.5) =$$

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 0 & 3.7688\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ 0 & -0.293839 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857\times10^{7} & -3.6057\times10^{5} \end{bmatrix}$$

Example: Cylinder Stresses

Finding the [*U*] matrix

Using the Forward Elimination Procedure of Gauss Elimination:
Step 1

$$\text{Row}4 - \left[\frac{\text{Row}1}{4.2857\times 10^{7}}\right]\times(0) =$$

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 0 & 3.7688\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ 0 & -0.293839 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857\times10^{7} & -3.6057\times10^{5} \end{bmatrix}$$

Example: Cylinder Stresses

Finding the [*U*] matrix

Using the Forward Elimination Procedure of Gauss Elimination:
Step 2

$$\text{Row3} - \left[\frac{\text{Row2}}{3.7688\times10^{5}}\right]\times(-0.293839) =$$

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 0 & 3.7688\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ 0 & 0 & -26.9140 & 0.579684 \\ 0 & 0 & 4.2857\times10^{7} & -3.6057\times10^{5} \end{bmatrix}$$

Example: Cylinder Stresses

Finding the [*U*] matrix

Using the Forward Elimination Procedure of Gauss Elimination:
Step 2

$$\text{Row4} - \left[\frac{\text{Row2}}{3.7688\times 10^{5}}\right]\times (0) =$$

$$\begin{bmatrix} 4.2857\times 10^{7} & -9.2307\times 10^{5} & 0 & 0 \\ 0 & 3.7688\times 10^{5} & -4.2857\times 10^{7} & 5.4619\times 10^{5} \\ 0 & 0 & -26.9140 & 0.579684 \\ 0 & 0 & 4.2857\times 10^{7} & -3.6057\times 10^{5} \end{bmatrix}$$

Example: Cylinder Stresses

Finding the [*U*] matrix

Using the Forward Elimination Procedure of Gauss Elimination: Step 3

$$\text{Row}4 - \left[\frac{\text{Row}3}{-26.9140}\right] \times (4.2857 \times 10^7) =$$

$$\begin{bmatrix} 4.2857\times10^7 & -9.2307\times10^5 & 0 & 0 \\ 0 & 3.7688\times10^5 & -4.2857\times10^7 & 5.4619\times10^5 \\ 0 & 0 & -26.9140 & 0.579684 \\ 0 & 0 & 0 & 5.62500\times10^5 \end{bmatrix}$$

Example: Cylinder Stresses

Finding the [*U*] matrix

Using the Forward Elimination Procedure of Gauss Elimination:

$$[U]=\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 0 & 3.7688\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ 0 & 0 & -26.9140 & 0.579684 \\ 0 & 0 & 0 & 5.62500\times10^{5} \end{bmatrix}$$

Example: Cylinder Stresses

Finding the [*L*] matrix

Using the multipliers from step 1 of the Forward Elimination Procedure

From the first step of forward elimination

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \ell_{21} & 1 & 0 & 0 \\ \ell_{31} & \ell_{32} & 1 & 0 \\ \ell_{41} & \ell_{42} & \ell_{43} & 1 \end{bmatrix}$$

$$\ell_{21} = \frac{a_{21}}{a_{11}} = \frac{4.2857\times 10^{7}}{4.2857\times 10^{7}} = 1$$

$$\ell_{31} = \frac{\mathrm{a}_{31}}{\mathrm{a}_{11}} = \frac{-6.5}{4.2857\times 10^{7}} = -1.51667\times 10^{-7}$$

$$\ell_{41} = \frac{a_{41}}{a_{11}} = \frac{0}{4.2857\times 10^{7}} = 0$$

Example: Cylinder Stresses

Finding the [*L*] matrix

Using the multipliers from step 2 of the Forward Elimination Procedure

From the second step of forward elimination

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \ell_{21} & 1 & 0 & 0 \\ \ell_{31} & \ell_{32} & 1 & 0 \\ \ell_{41} & \ell_{42} & \ell_{43} & 1 \end{bmatrix}$$

$$\ell_{32} = \frac{\mathrm{a}_{32}}{\mathrm{a}_{22}} \frac{-0.293839}{3.7688\times 10^{5}} = -7.79662\times 10^{-7}$$

$$\ell_{42} = \frac{a_{42}}{a_{22}} = \frac{0}{3.7688\times 10^{5}} = 0$$

Example: Cylinder Stresses

Finding the [*L*] matrix

Using the multipliers from step 3 of the Forward Elimination Procedure

From the third step of forward elimination

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \ell_{21} & 1 & 0 & 0 \\ \ell_{31} & \ell_{32} & 1 & 0 \\ \ell_{41} & \ell_{42} & \ell_{43} & 1 \end{bmatrix} \quad \ell_{43} = \frac{a_{43}}{a_{33}} = \frac{4.2857\times10^{7}}{-26.9140} = -1.59237\times10^{6}$$

$$[L] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1.51667\times10^{-7} & -7.79662\times10^{-7} & 1 & 0 \\ 0 & 0 & -1.59237\times10^{6} & 1 \end{bmatrix}$$

Example: Cylinder Stresses

$$[L] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1.51667\times10^{-7} & -7.79662\times10^{-7} & 1 & 0 \\ 0 & 0 & -1.59237\times10^{6} & 1 \end{bmatrix}$$

Does $[L][U]=[A]$?

$$[L][U]=$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1.51667\times10^{-7} & -7.79662\times10^{-7} & 1 & 0 \\ 0 & 0 & -1.59237\times10^{6} & 1 \end{bmatrix} \begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 0 & 3.7688\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ 0 & 0 & -26.9140 & 0.579684 \\ 0 & 0 & 0 & 5.62500\times10^{5} \end{bmatrix}$$

Example: Cylinder Stresses

Example: Solving simultaneous linear equations using LU Decomposition

Set $[L][Z]=[C]$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1.51667\times10^{-7} & -7.79662\times10^{-7} & 1 & 0 \\ 0 & 0 & -1.59237\times10^{6} & 1 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}=\begin{bmatrix} -7.887\times10^{3} \\ 0 \\ 0.007 \\ 0 \end{bmatrix}$$

$$z_1 = -7.887\times10^{3}$$

$$z_1 + z_2 = 0$$

Solve for $[Z]$

$$-1.51667\times10^{-7} z_1 + (-7.79662\times10^{-7}) z_2 + z_3 = 0.007$$

$$-1.59237\times10^{6} z_3 + z_4 = 0$$

Example: Cylinder Stresses

Example: Solving simultaneous linear equations using LU Decomposition

Complete the forward substitution to solve for $[Z]$

$$z_1 = -7.887 \times 10^3$$

$$\begin{aligned} z_2 &= -z_1 \\ &= -(-7.887 \times 10^3) \\ &= 7.887 \times 10^3 \end{aligned}$$

$$z_3 = 0.007 - (-1.51667 \times 10^{-7}) z_1 - (-7.79662 \times 10^{-7}) z_2$$

$$= 0.007 - (-1.51667 \times 10^{-7}) \times (-7.887 \times 10^3) - (-7.79662 \times 10^{-7}) \times (7.887 \times 10^3)$$

$$= 1.19530 \times 10^{-2}$$

Example: Cylinder Stresses

Example: Solving simultaneous linear equations using LU Decomposition

Complete the forward substitution to solve for $[Z]$

$$
\begin{aligned}
z_4 &= -(-1.59237\times10^{6})z_3 \\
&= -(-1.59237\times10^{6})\times(1.19530\times10^{-2}) \\
&= 19033.6
\end{aligned}
$$

$$
[Z]=\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}=\begin{bmatrix} -7.887\times10^{3} \\ 7.887\times10^{3} \\ 1.19530\times10^{-2} \\ 19033.6 \end{bmatrix}
$$

Example: Cylinder Stresses

Example: Solving simultaneous linear equations using LU Decomposition

Set $[U][X]=[Z]$

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 0 & 3.7688\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ 0 & 0 & -26.9140 & 0.579684 \\ 0 & 0 & 0 & 5.62500\times10^{5} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -7.887\times10^{3} \\ 7.887\times10^{3} \\ 1.19530\times10^{-2} \\ 19033.6 \end{bmatrix}$$

The four equations become:

$$4.2857\times10^{7}x_1 + (-9.2307\times10^{5})x_2 + (0)x_3 + (0)x_4 = -7.887\times10^{3}$$

$$3.7668\times10^{5}x_2 + (-4.2857\times10^{7})x_3 + 5.4619\times10^{5}x_4 = 7.887\times10^{3}$$

$$-26.9140x_3 + 0.579684x_4 = 1.19530\times10^{-2}$$

$$5.62500\times10^{5}x_4 = 19033.6$$

Example: Cylinder Stresses

Example: Solving simultaneous linear equations using LU Decomposition

From the 4th equation

$$5.62500\times10^{5}x_4 = 19033.6$$

$$x_4 = \frac{19033.6}{5.62500\times10^{5}}$$

$$= 3.38375\times10^{-2}$$

Substituting in x_4 and using the third equation

$$-26.9140x_3 + 0.579684x_4 = 1.19530\times10^{-2}$$

$$x_3 = \frac{1.19530\times10^{-2} - 0.579684x_4}{-26.9140}$$

$$= \frac{1.19530\times10^{-2} - 0.579684\times(3.38375\times10^{-2})}{-26.9140}$$

$$= 2.84687\times10^{-4}$$

Example: Cylinder Stresses

Example: Solving simultaneous linear equations using LU Decomposition

Substituting in x_4 and x_3 into the second equation

$$3.7668\times10^{5}x_2+(-4.2857\times10^{7})x_3+5.4619\times10^{5}x_4=7.887\times10^{3}$$

$$x_2=\frac{7.887\times10^{3}-(-4.2857\times10^{7})x_3-5.4619\times10^{5}x_4}{3.7668\times10^{5}}$$

$$=\frac{7.887\times10^{3}-(-4.2857\times10^{7})\times(2.84687\times10^{-4})-5.4619\times10^{5}\times(3.38375\times10^{-2})}{3.7668\times10^{5}}$$

$$=4.26390\times10^{-3}$$

Example: Cylinder Stresses

Example: Solving simultaneous linear equations using LU Decomposition

Substituting in x_4, x_3 and x_2 into the first equation

$$4.2857\times10^{7}x_1+(-9.2307\times10^{5})x_2+(0)x_3+(0)x_4=-7.887\times10^{3}$$

$$x_1=\frac{-7.887\times10^{3}-(-9.2307\times10^{5})x_2-(0)x_3-(0)x_4}{4.2857\times10^{7}}$$

$$=\frac{-7.887\times10^{3}-(-9.2307\times10^{5})\times(4.26390\times10^{-3})}{4.2857\times10^{7}}$$

$$=9.21931\times10^{-5}$$

Example: Cylinder Stresses

Solution:

The solution vector is

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -9.21931\times10^{-5} \\ 4.26390\times10^{-3} \\ 2.84687\times10^{-4} \\ 3.38375\times10^{-2} \end{bmatrix}$$

LU Decomposition

Finding the inverse of a square matrix

Remember, the relative computational time comparison of LU decomposition and Gauss elimination is:

$$\frac{n^4}{3} + \frac{n^3}{2} \rangle\rangle \frac{4n^3}{3}$$

Review: The inverse [B] of a square matrix [*A*] is defined as

$$[A][B] = [I] = [B][A]$$

LU Decomposition

Finding the inverse of a square matrix

How can LU Decomposition be used to find the inverse?

Assume the first column of [*B*] to be $\begin{bmatrix} b_{11} & b_{12} & \ldots & b_{n1} \end{bmatrix}^T$

Using this and the definition of matrix multiplication

First column of [*B*]

$$[A]\begin{bmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{n1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Second column of [*B*]

$$[A]\begin{bmatrix} b_{12} \\ b_{22} \\ \vdots \\ b_{n2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

The remaining columns in [*B*] can be found in the same manner

LU Decomposition

Example: Finding the inverse of a square matrix

Find the inverse of [*A*]

$$[A]=\begin{bmatrix}25 & 5 & 1\\ 64 & 8 & 1\\ 144 & 12 & 1\end{bmatrix}$$

Using the Decomposition procedure, the [*L*] and [*U*] matrices are found to be

$$[A]=[L][U]=\begin{bmatrix}1 & 0 & 0\\ 2.56 & 1 & 0\\ 5.76 & 3.5 & 1\end{bmatrix}\begin{bmatrix}25 & 5 & 1\\ 0 & -4.8 & -1.56\\ 0 & 0 & 0.7\end{bmatrix}$$

LU Decomposition

Example: Finding the inverse of a square matrix

Solving for the each column of [*B*] requires to steps

1) Solve [*L*] [*Z*] = [*C*] for [*Z*] and 2) Solve [*U*] [*X*] = [*Z*] for [*X*]

Step 1: $[\mathrm{L}][\mathrm{Z}]=[\mathrm{C}] \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

This generates the equations:

$$z_1 = 1$$

$$2.56 z_1 + z_2 = 0$$

$$5.76 z_1 + 3.5 z_2 + z_3 = 0$$

LU Decomposition

Example: Finding the inverse of a square matrix

Solving for $[Z]$

$$z_1 = 1$$

$$\begin{aligned} z_2 &= 0 - 2.56 z_1 \\ &= 0 - 2.56(1) \\ &= -2.56 \end{aligned}$$

$$\begin{aligned} z_3 &= 0 - 5.76 z_1 - 3.5 z_2 \\ &= 0 - 5.76(1) - 3.5(-2.56) \\ &= 3.2 \end{aligned}$$

$$[Z] = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2.56 \\ 3.2 \end{bmatrix}$$

LU Decomposition

Example: Finding the inverse of a square matrix

Solving for [*U*] [*X*] = [*Z*] for [*X*]

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ -2.56 \\ 3.2 \end{bmatrix}$$

$$25b_{11} + 5b_{21} + b_{31} = 1$$

$$-4.8b_{21} - 1.56b_{31} = -2.56$$

$$0.7b_{31} = 3.2$$

LU Decomposition

Example: Finding the inverse of a square matrix

Using Backward Substitution

$$b_{31} = \frac{3.2}{0.7} = 4.571$$

$$b_{21} = \frac{-2.56 + 1.560 b_{31}}{-4.8}$$

$$= \frac{-2.56 + 1.560(4.571)}{-4.8} = -0.9524$$

$$b_{11} = \frac{1 - 5b_{21} - b_{31}}{25}$$

$$= \frac{1 - 5(-0.9524) - 4.571}{25} = 0.04762$$

So the first column of the inverse of [*A*] is:

$$\begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} = \begin{bmatrix} 0.04762 \\ -0.9524 \\ 4.571 \end{bmatrix}$$

LU Decomposition

Example: Finding the inverse of a square matrix

Repeating for the second and third columns of the inverse

Second Column

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}\begin{bmatrix} b_{12} \\ b_{22} \\ b_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} b_{12} \\ b_{22} \\ b_{32} \end{bmatrix} = \begin{bmatrix} -0.08333 \\ 1.417 \\ -5.000 \end{bmatrix}$$

Third Column

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}\begin{bmatrix} \mathrm{b}_{13} \\ \mathrm{b}_{23} \\ \mathrm{b}_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} b_{13} \\ b_{23} \\ b_{33} \end{bmatrix} = \begin{bmatrix} 0.03571 \\ -0.4643 \\ 1.429 \end{bmatrix}$$

LU Decomposition

Example: Finding the inverse of a square matrix

The inverse of [*A*] is

$$[A]^{-1} = \begin{bmatrix} 0.4762 & 0.08333 & 0.0357 \\ -0.9524 & 1.417 & -0.4643 \\ 4.571 & -5.050 & 1.429 \end{bmatrix}$$

To check your work do the following operation

$$[A][A]^{-1} = [I] = [A]^{-1}[A]$$