

# Roots of a Nonlinear Equation

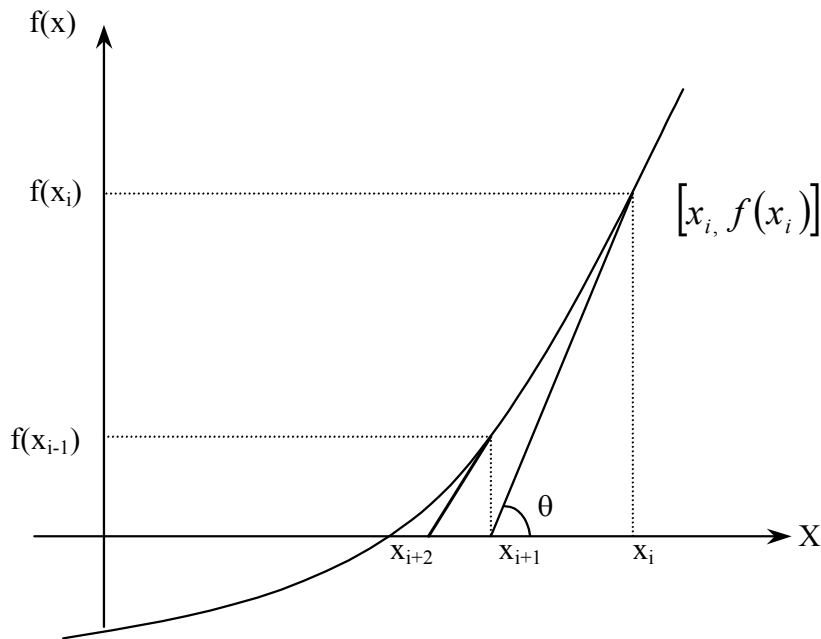


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Topic: Secant Method

Major: Computer Engineering

# Secant Method



## Newton's Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

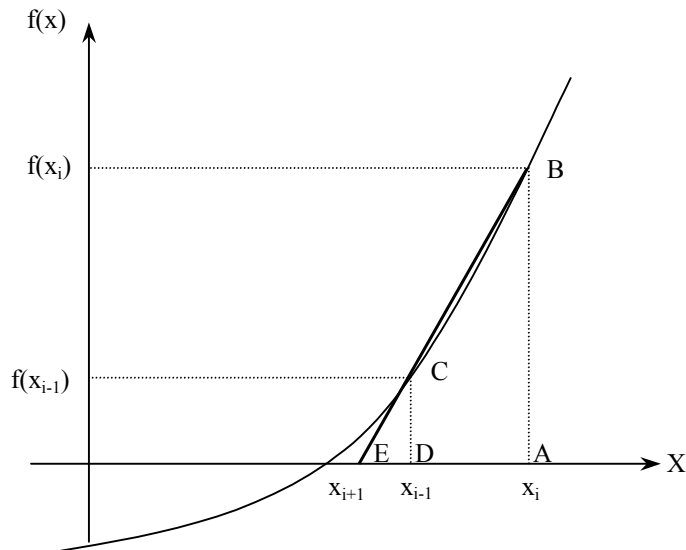
Approximate the derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

# Secant Method

## Geometric Similar Triangles



$$\frac{AB}{AE} = \frac{DC}{DE}$$

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$



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# Algorithm for Secant Method



# Step 1

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Calculate the next estimate of the root from two initial guesses

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Find the absolute relative approximate error

$$|\epsilon_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$$



## Step 2

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Find if the absolute relative approximate error is greater than the prespecified relative error tolerance.

If so, go back to step 1, else stop the algorithm.

Also check if the number of iterations has exceeded the maximum number of iterations.



# Example

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- To find the inverse of a number 'a', one can use the equation

$$f(x) = a - \frac{1}{x} = 0$$

where  $x$  is the inverse of 'a'.



# Solution

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Use the Secant method of finding roots of equations to

- Find the inverse of  $a = 2.5$ . Conduct three iterations to estimate the root of the above equation.
- Find the absolute relative approximate error at the end of each iteration, and
- The number of significant digits at least correct at the end of each iteration.

# Graph of function $f(x)$

$$f(x) = a - \frac{1}{x} = 0$$

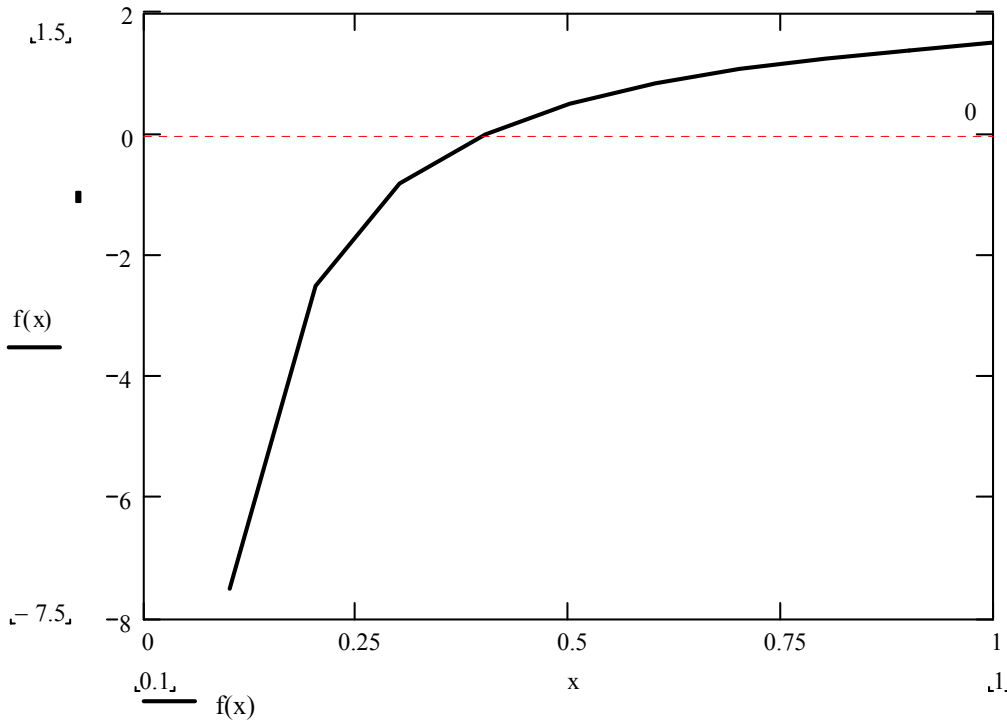
$$x_{i+1} = x_i - \frac{\left(a - \frac{1}{x_i}\right)(x_i - x_{i-1})}{\left(a - \frac{1}{x_i}\right) - \left(a - \frac{1}{x_{i-1}}\right)}$$

$$= x_i - \frac{\left(a - \frac{1}{x_i}\right)(x_i - x_{i-1})}{\frac{1}{x_{i-1}} - \frac{1}{x_i}}$$

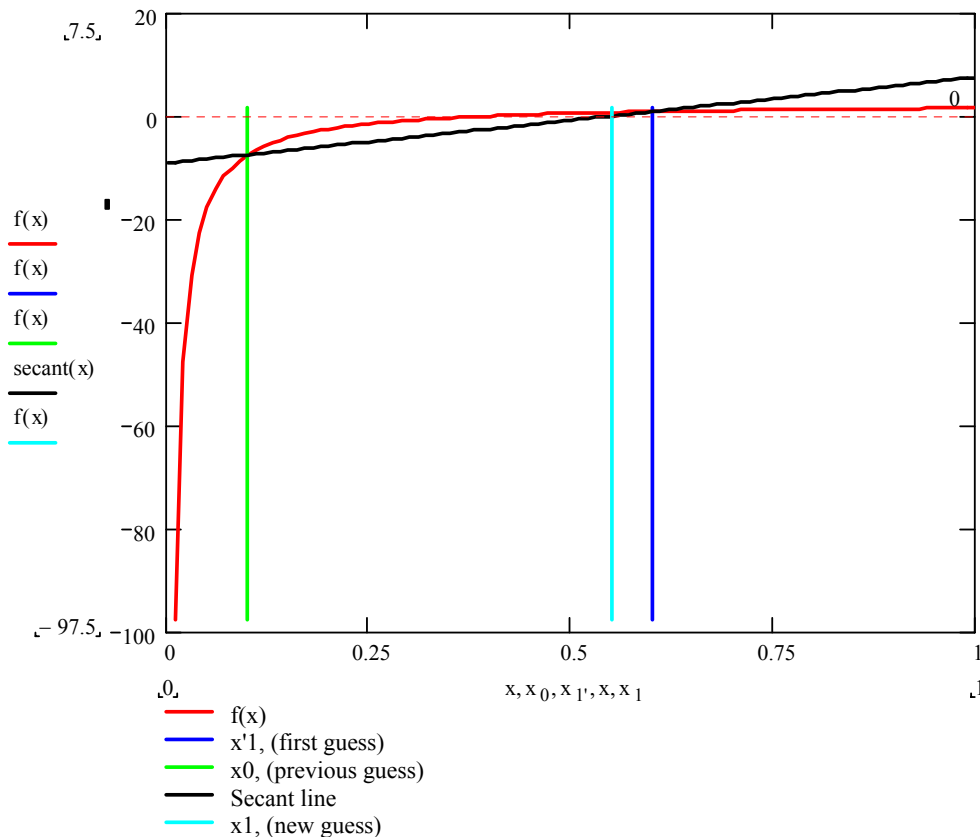
$$= x_i - \frac{\left(a - \frac{1}{x_i}\right)(x_i - x_{i-1})}{\frac{x_i - x_{i-1}}{x_i x_{i-1}}}$$

$$= x_i - x_i x_{i-1} \left(a - \frac{1}{x_i}\right)$$

$$\therefore x_{i+1} = x_i - x_{i-1} (ax_i - 1)$$



# Iteration #1



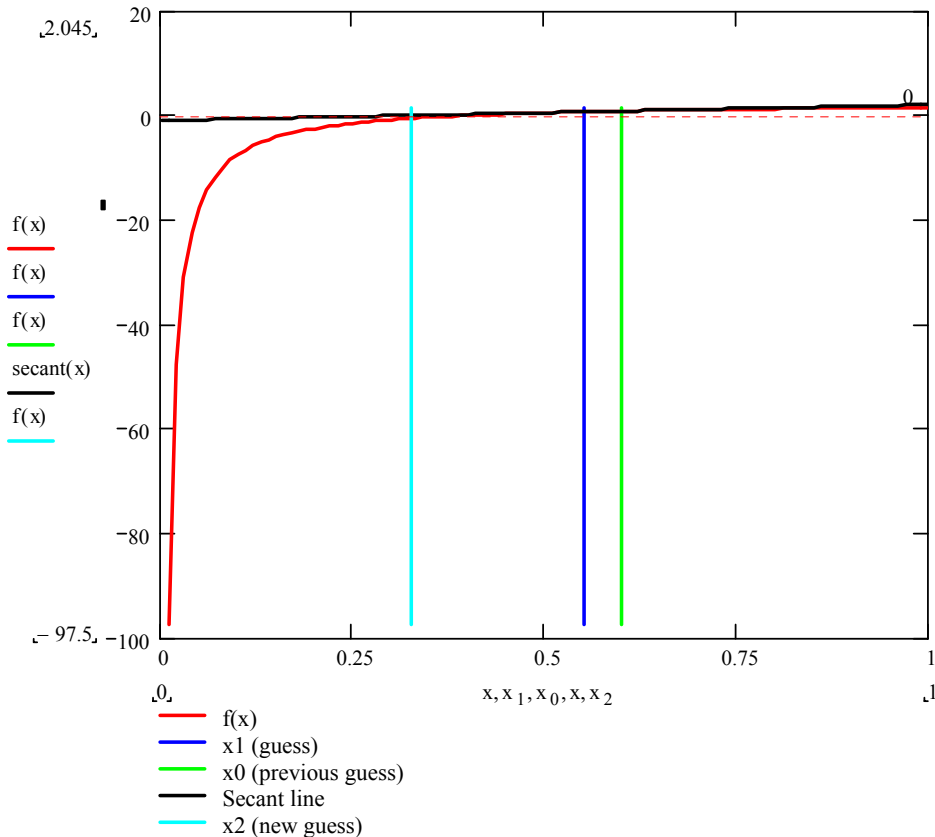
$$x_{-1} = 0.1, x_0 = 0.6$$

$$x_1 = x_0 - x_{-1}(ax_0 - 1)$$

$$x_1 = 0.6 - 0.1(2.5(0.6) - 1) = 0.55$$

$$|\epsilon_a| = 9.091\%$$

# Iteration #2



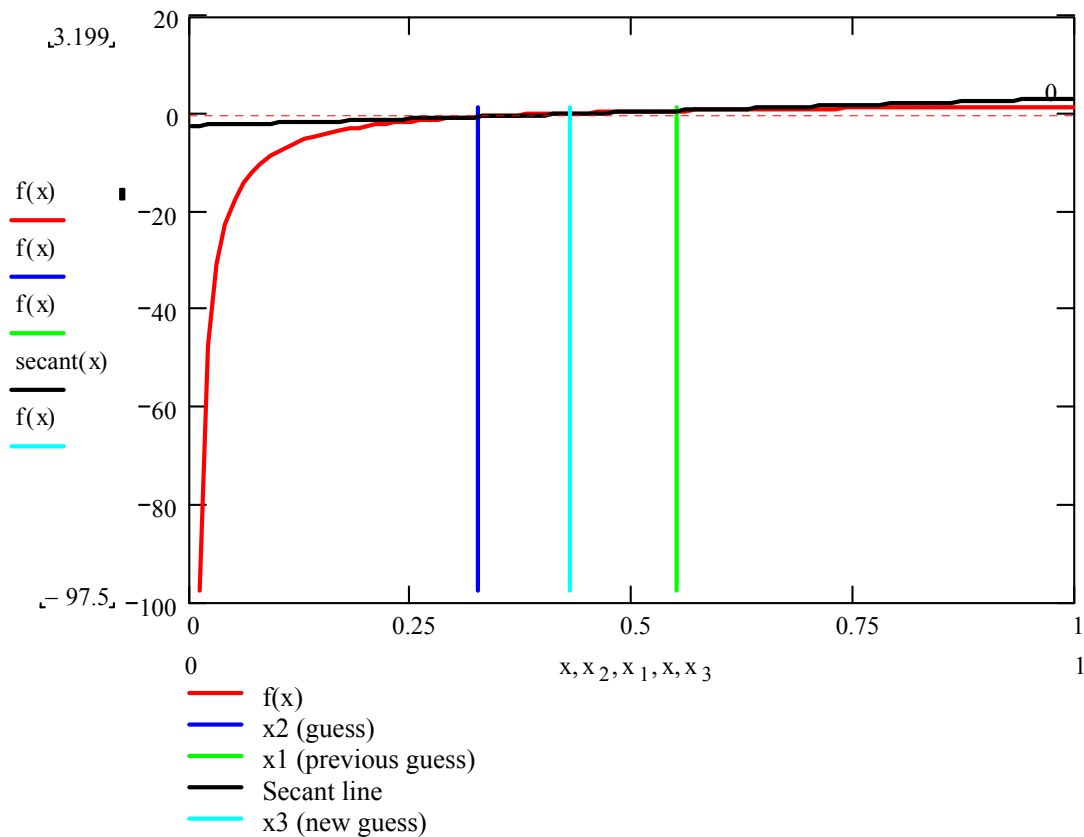
$$x_0 = 0.6, x_1 = 0.55$$

$$x_2 = x_1 - x_0(ax_1 - 1)$$

$$x_2 = 0.55 - 0.6(2.5(0.55) - 1) = 0.325$$

$$|\epsilon_a| = 69.231\%$$

# Iteration #3



$$x_1 = 0.55, x_2 = 0.325$$

$$x_3 = x_2 - x_1(ax_2 - 1)$$

$$x_3 = 0.325 - 0.55(2.5(0.325 - 1))$$

$$= 0.428$$

$$|\epsilon_a| = 24.0876 \%$$

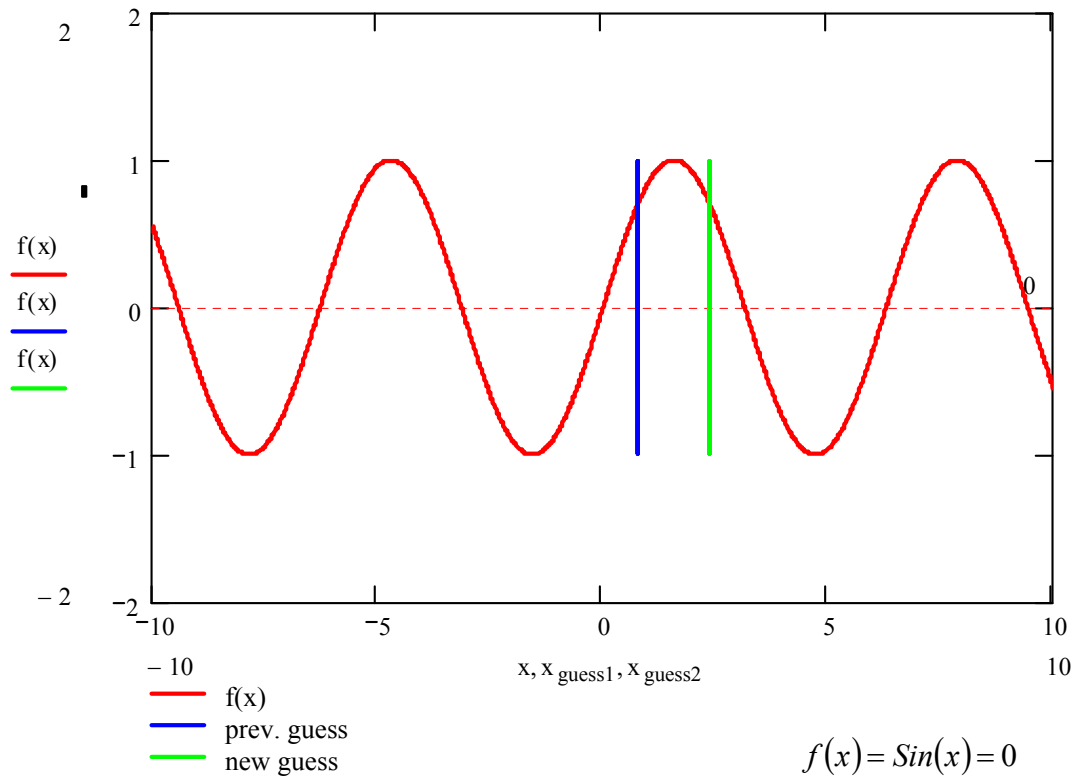


# Advantages

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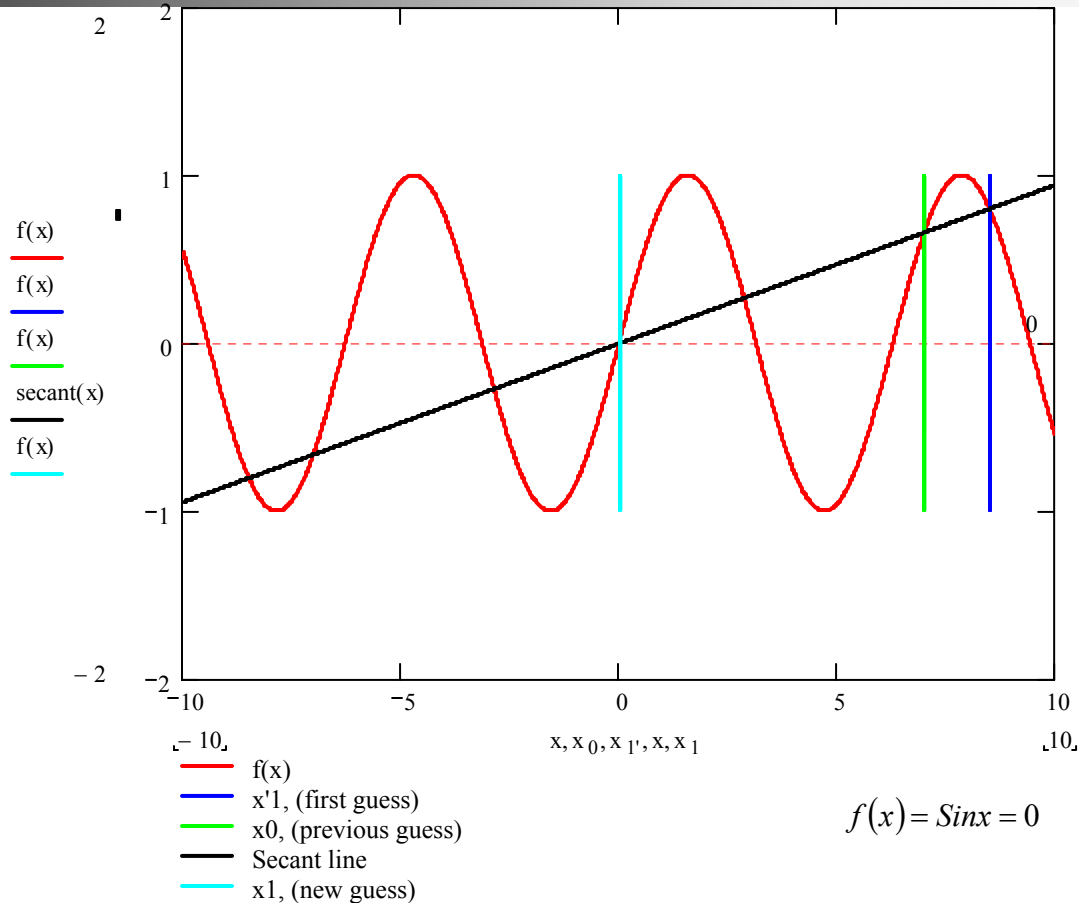
- Converges fast, if it converges
- Requires two guesses that do not need to bracket the root

# Drawbacks



Division by zero

# Drawbacks (continued)



## Root Jumping