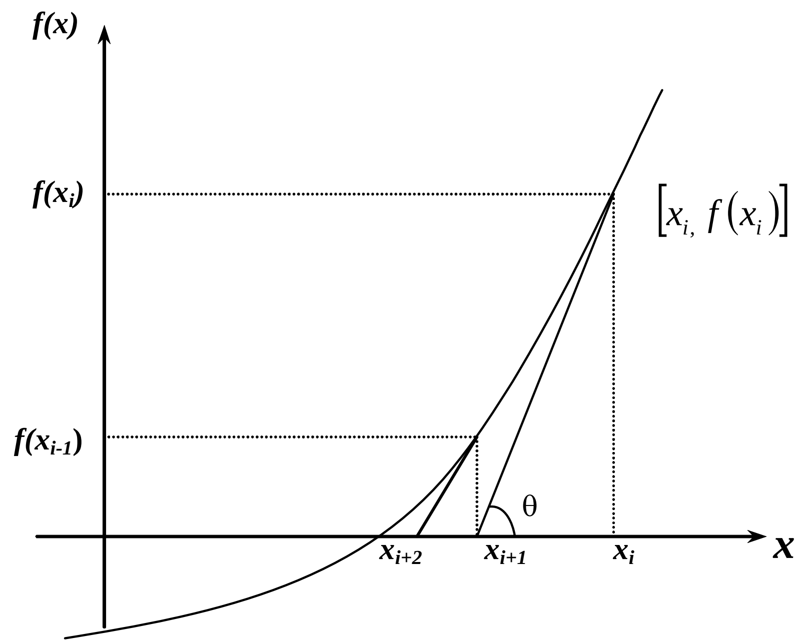
Roots of a Nonlinear Equation

Topic: Newton-Raphson Method

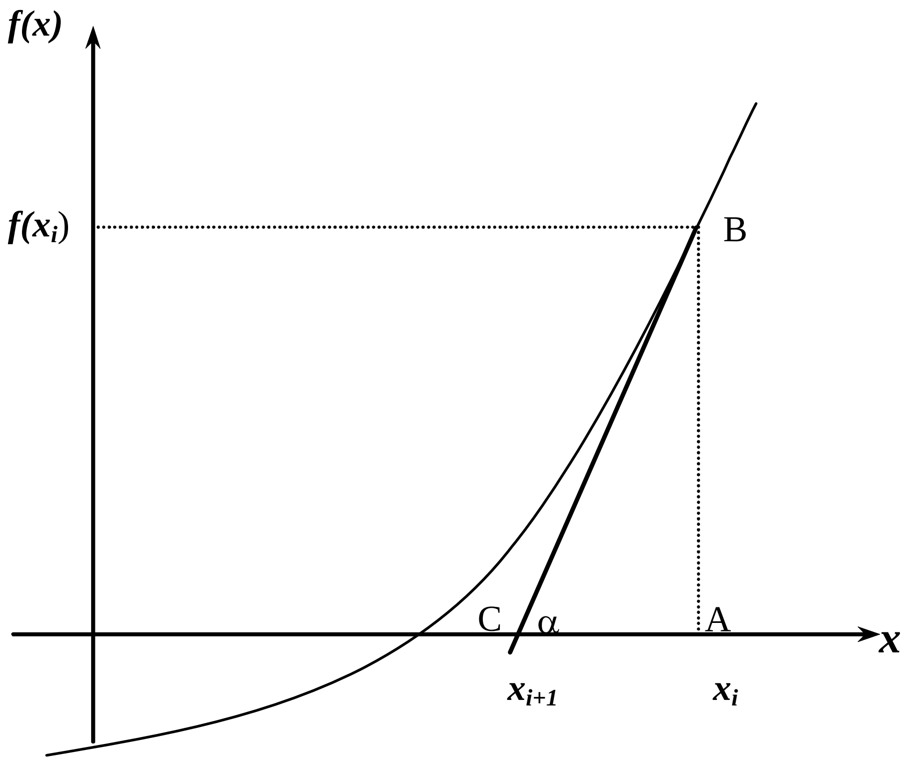
Major: Electrical Engineering

Newton-Raphson Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$



Derivation



$$\tan(\alpha) = \frac{AB}{AC}$$

$$f'(x_i) = \frac{f(x_i)}{x_i - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Algorithm for Newton-Raphson Method

Step 1

Evaluate $f'(x)$ symbolically

Step 2

Calculate the next estimate of the root

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Find the absolute relative approximate error

$$|\in_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$$

Step 3

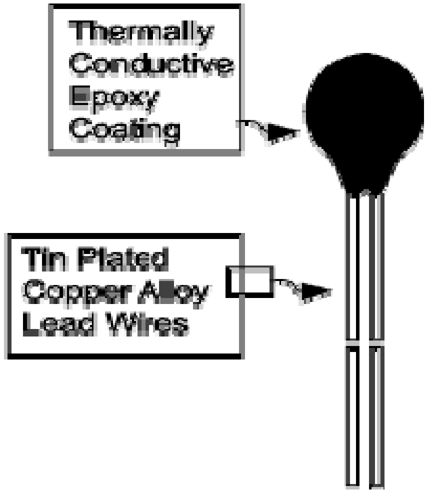
- Find if the absolute relative approximate error is greater than the pre-specified relative error tolerance.

- If so, go back to step 2, else stop the algorithm.

- Also check if the number of iterations has exceeded the maximum number of iterations.

Example

- Thermistors are temperature-measuring devices based on the principle that the thermistor material exhibits a change in electrical resistance with a change in temperature. By measuring the resistance of the thermistor material, one can then determine the temperature.



Solution

For a 10K3A Betatherm thermistor, the relationship between the resistance 'R' of the thermistor and the temperature is given by

$$\frac{1}{T} = 1.129241\times10^{-3} + 2.341077\times10^{-4}\ln(R) + 8.775468\times10^{-8}\{\ln(R)\}^3$$

where note that T is in Kelvin and R is in ohms.

Solution

For the thermistor error of no more than +-0.01^{o}C is acceptable. To find the range of the resistance that is within this acceptable limit at 19^{o}C, we need to solve

$$\frac{1}{19.01+273.15}=1.129241\times10^{-3}+2.341077\times10^{-4}\ln(R)+8.775468\times10^{-8}\{\ln(R)\}^{3}$$

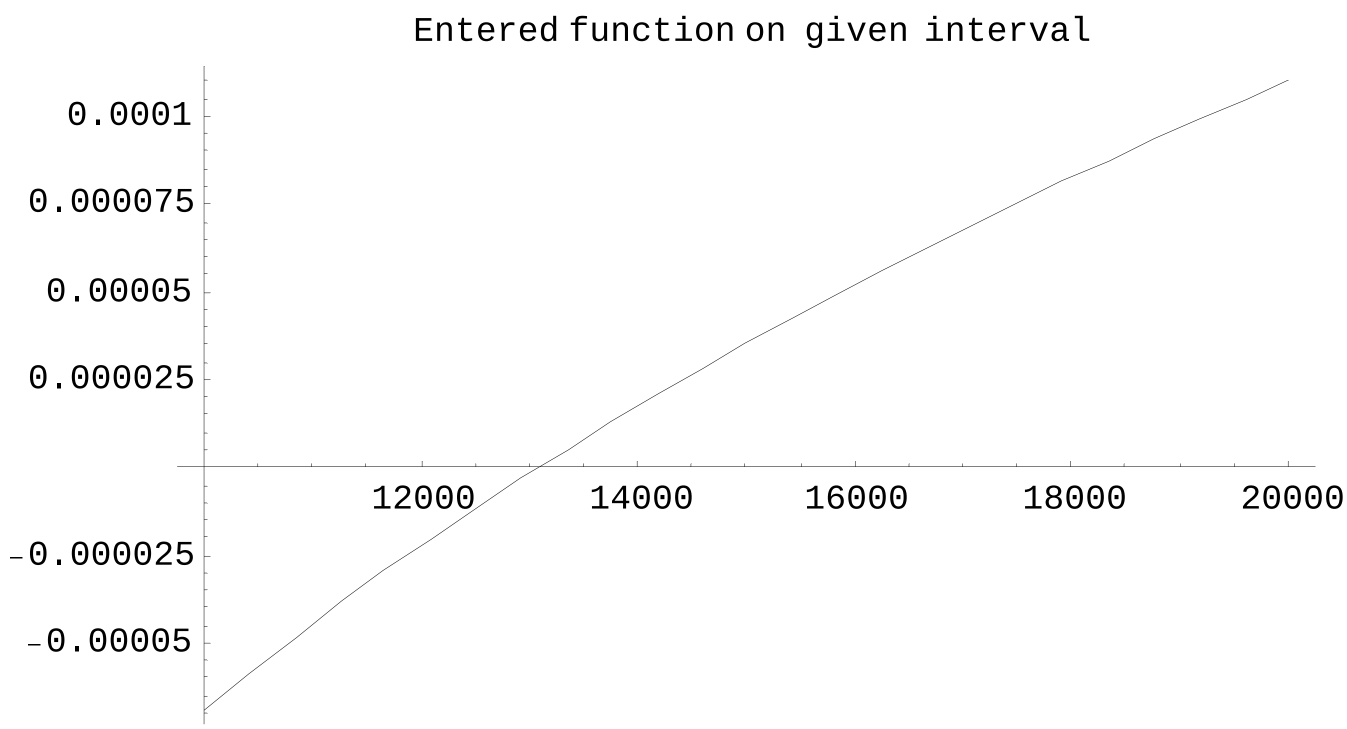
and

$$\frac{1}{18.99+273.15}=1.129241\times10^{-3}+2.341077\times10^{-4}\ln(R)+8.775468\times10^{-8}\{\ln(R)\}^{3}$$

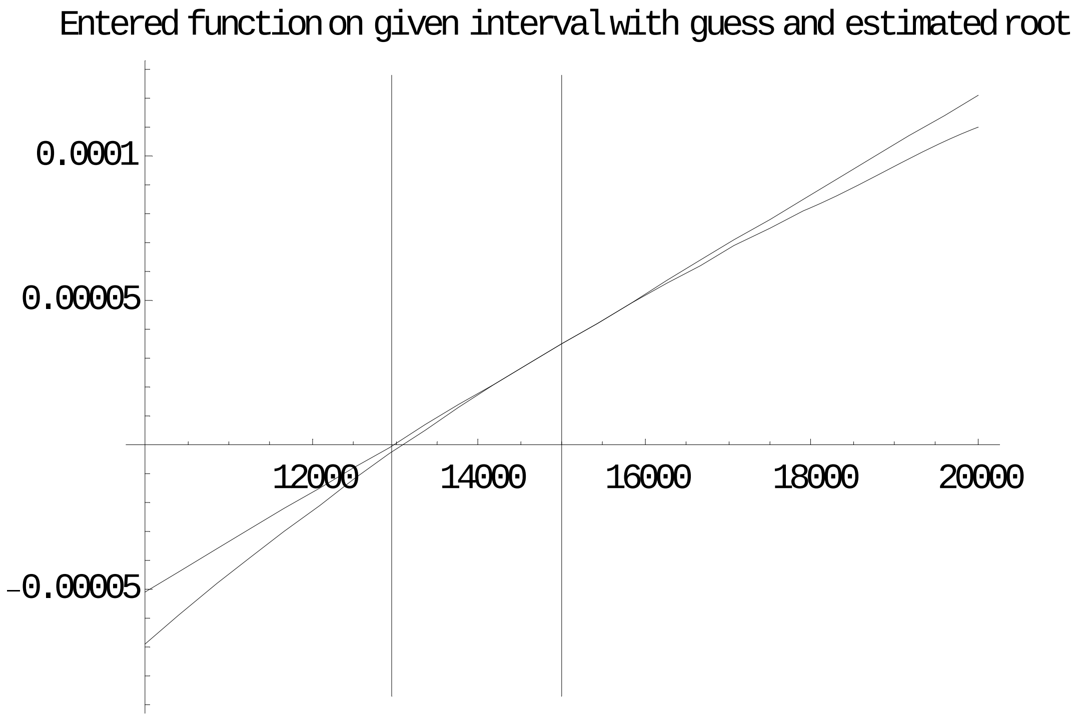
Use the Newton-Raphson method of finding roots of equations to find the resistance R at 18.99^{o}C. Conduct three iterations to estimate the root of the above equation.

Graph of function f(x)

$$f(R) = 2.341077\times10^{-4}\ln(R) + 8.775468\times10^{-8}\{\ln(R)\}^3 - 2.293775\times10^{-3}$$



Iteration #1



$$R_0 = 15000$$

$$R_1 = R_0 - \frac{f(R_0)}{f'(R_0)}$$

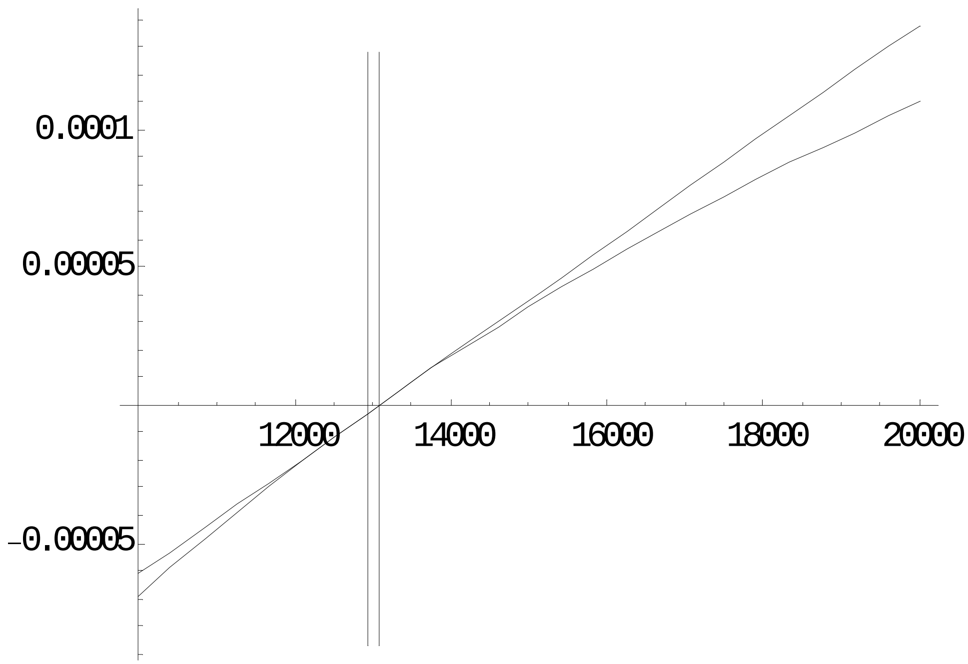
$$R_1 = 2 - \frac{3.53829 \times 10^{-5}}{1.723 \times 10^{-8}}$$

$$= 12946.435$$

$$|\epsilon_a| = 15.862\%$$

Iteration #2

Entered function on given interval with guess and estimated root



$$R_1 = 12946.435$$

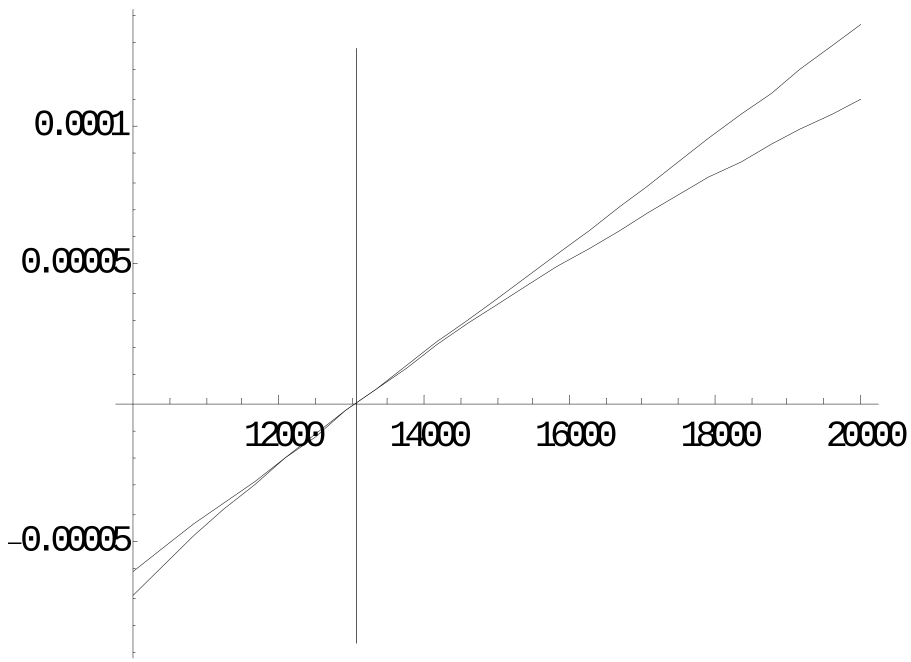
$$R_2 = R_1 - \frac{f(R_1)}{f'(R_1)}$$

$$R_2 = 12946.435 - \frac{-2.6140 \times 10^{-6}}{1.99059 \times 10^{-8}}$$

$$= 13077.753$$

$$\left|\in_a\right| = 1.0041\,\%$$

Iteration #3



$$R_2 = 13077.753$$

$$R_3 = R_2 - \frac{f(R_2)}{f'(R_2)}$$

$$= 13077.753 - \frac{-1.29143 \times 10^{-8}}{1.97099 \times 10^{-8}}$$

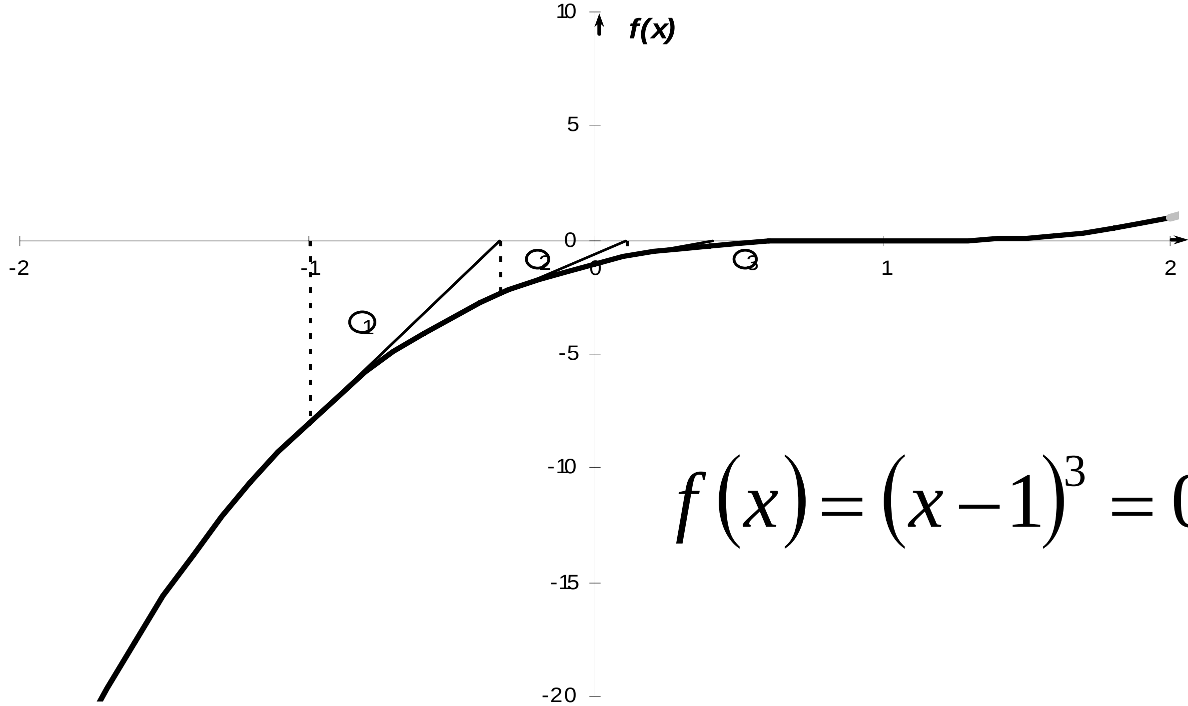
$$= 13078.408$$

$$|\in_a| = 0.0050082\ \%$$

Advantages

- Converges fast, if it converges
- Requires only one guess

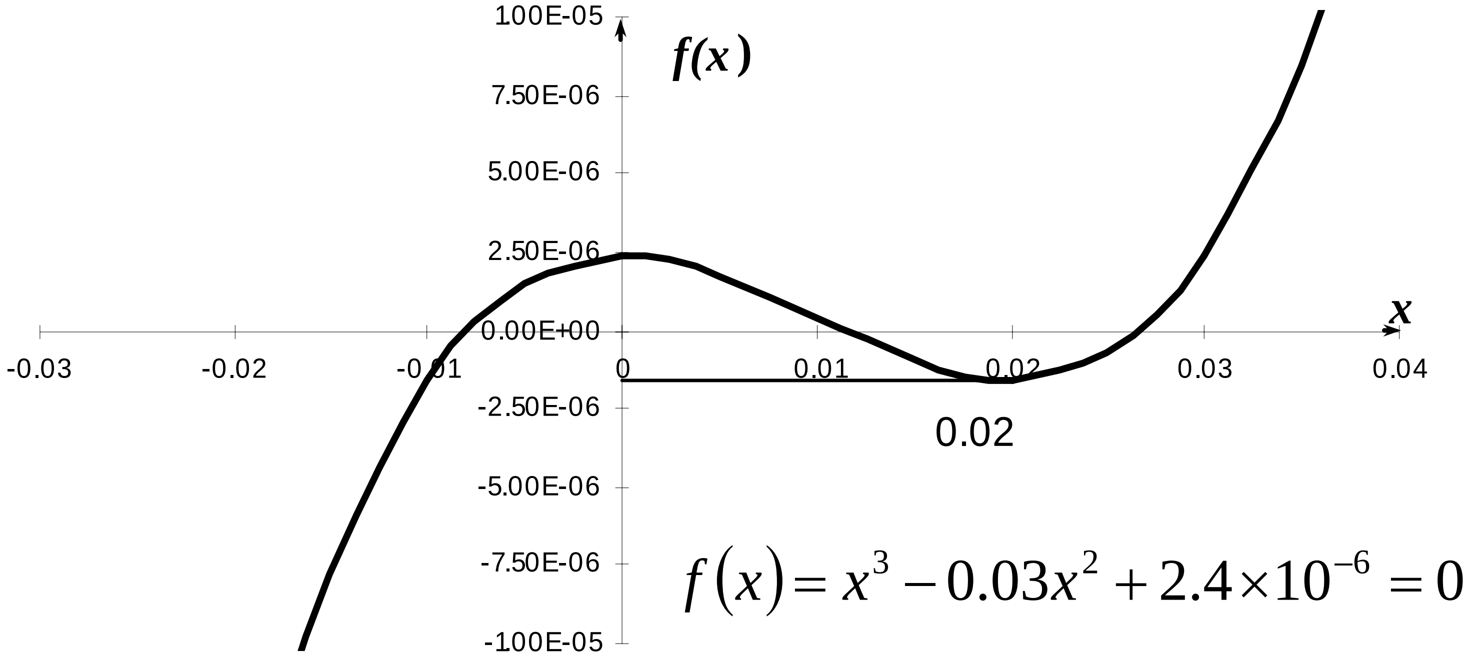
Drawbacks



$$f(x) = (x-1)^3 = 0$$

Inflection Point

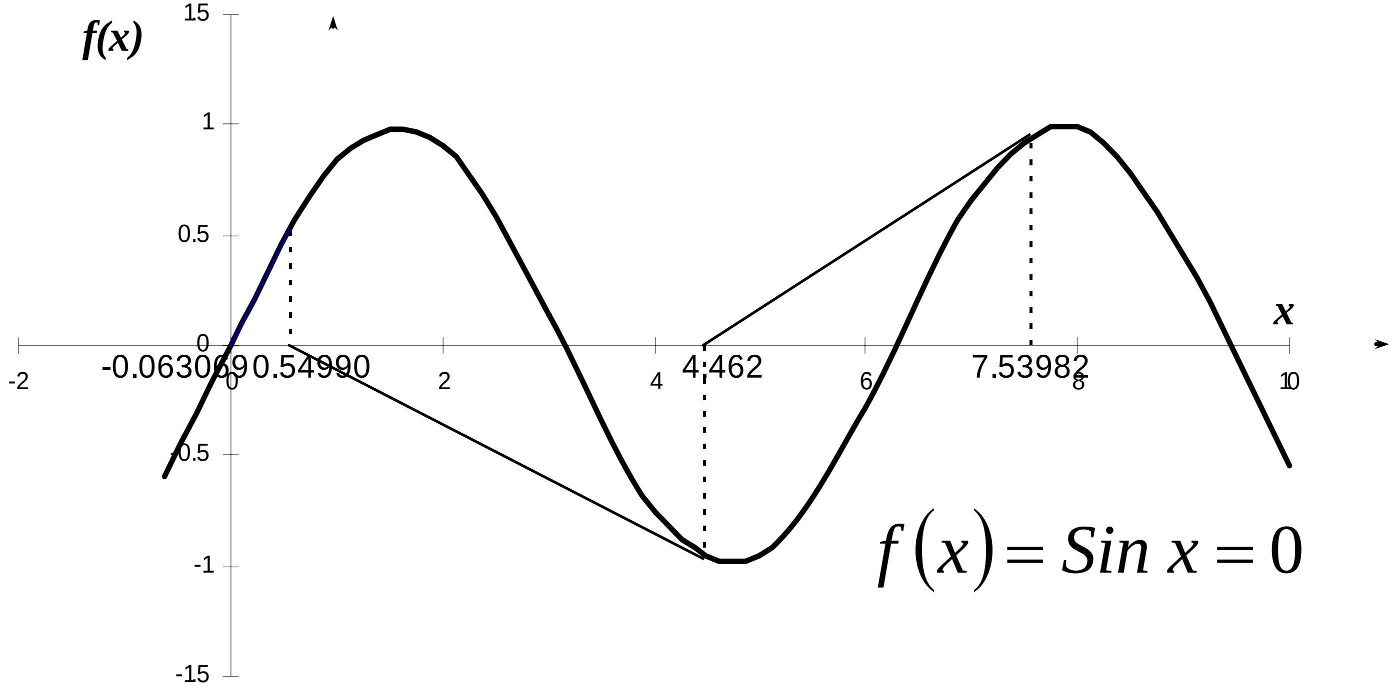
Drawbacks (continued)



$$f(x) = x^3 - 0.03x^2 + 2.4\times10^{-6} = 0$$

Division by zero

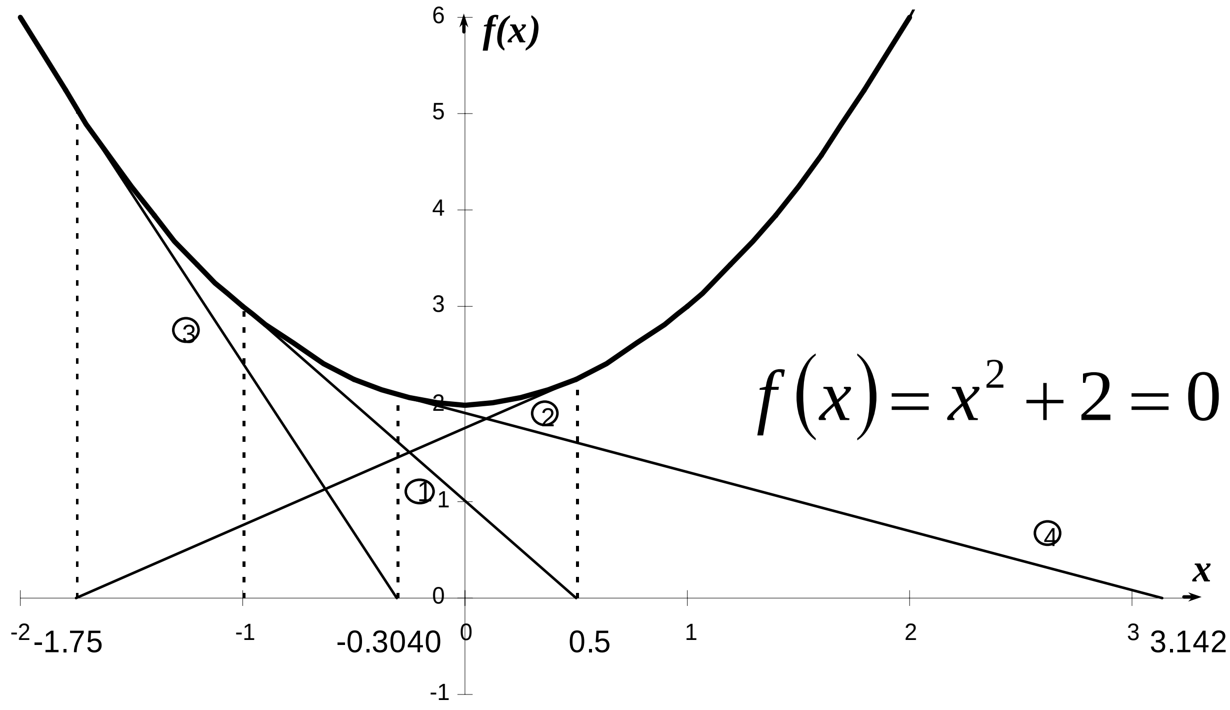
Drawbacks (continued)



$$f(x) = Sin\ x = 0$$

Root Jumping

Drawbacks (continued)



$$f(x) = x^2 + 2 = 0$$

Oscillations near Local Maxima or Minima