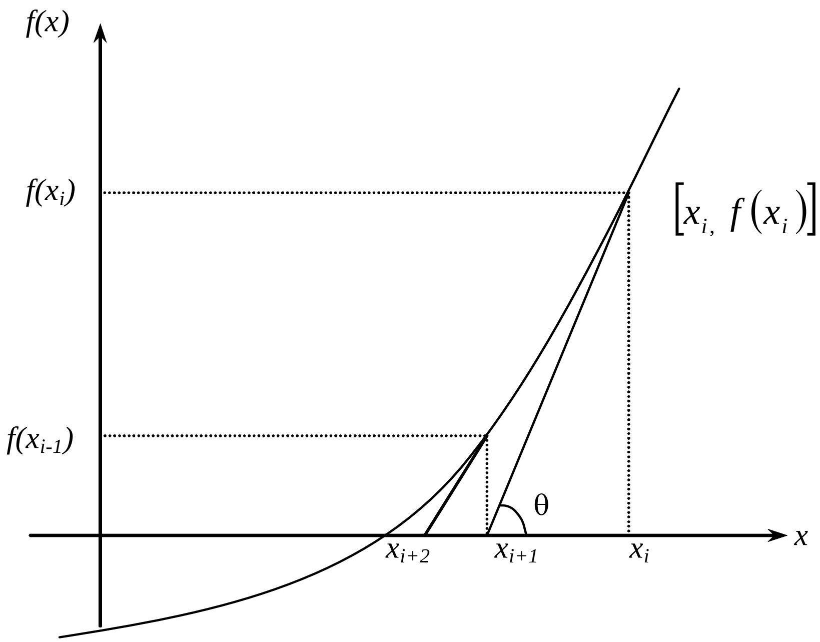
Roots of a Nonlinear Equation

Topic: Secant Method

Major: Electrical Engineering

Secant Method



Newton's Method

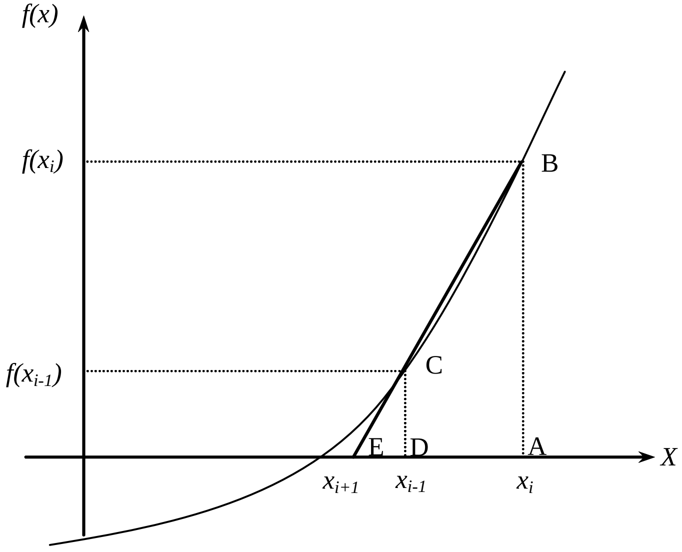
$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Approximate the derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Secant Method



Geometric Similar Triangles

$$\frac{AB}{AE} = \frac{DC}{DE}$$

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Algorithm for Secant Method

Step 1

Calculate the next estimate of the root from two initial guesses

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Find the absolute relative approximate error

$$|\in_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$$

Step 2

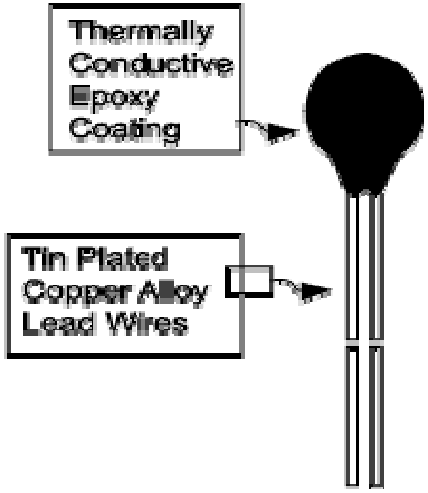
Find if the absolute relative approximate error is greater than the prespecified relative error tolerance.

If so, go back to step 1, else stop the algorithm.

Also check if the number of iterations has exceeded the maximum number of iterations.

Example

- Thermistors are temperature-measuring devices based on the principle that the thermistor material exhibits a change in electrical resistance with a change in temperature. By measuring the resistance of the thermistor material, one can then determine the temperature.



Solution

For a 10K3A Betatherm thermistor, the relationship between the resistance 'R' of the thermistor and the temperature is given by

$$\frac{1}{T} = 1.129241\times10^{-3} + 2.341077\times10^{-4}\ln(R) + 8.775468\times10^{-8}\{\ln(R)\}^3$$

where note that T is in Kelvin and R is in ohms.

Solution

For the thermistor error of no more than +-0.01^{o}C is acceptable. To find the range of the resistance that is within this acceptable limit at 19^{o}C, we need to solve

$$\frac{1}{19.01+273.15}=1.129241\times10^{-3}+2.341077\times10^{-4}\ln(R)+8.775468\times10^{-8}\{\ln(R)\}^{3}$$

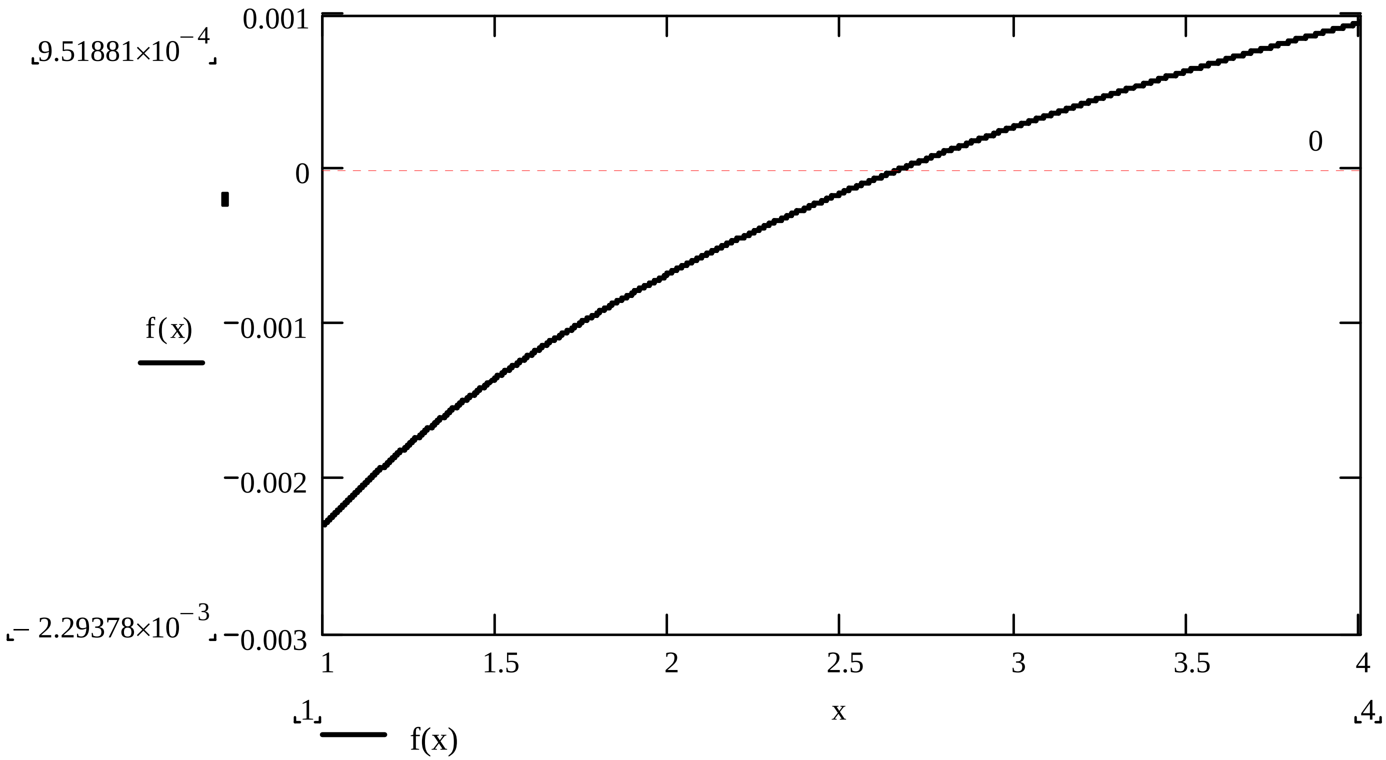
and

$$\frac{1}{18.99+273.15}=1.129241\times10^{-3}+2.341077\times10^{-4}\ln(R)+8.775468\times10^{-8}\{\ln(R)\}^{3}$$

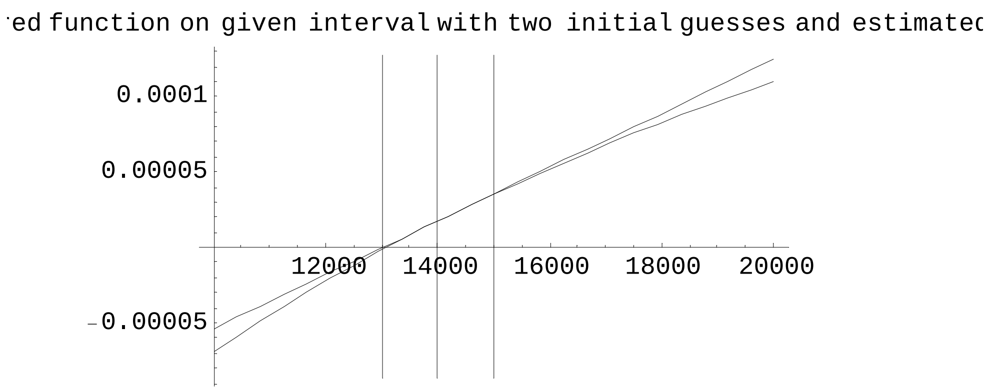
Use the Secant method of finding roots of equations to find the resistance R at 18.99^{o}C. Conduct three iterations to estimate the root of the above equation.

Graph of function f(x)

$$f(R) = 2.341077\times10^{-4}\ln(R) + 8.775468\times10^{-8}\{\ln(R)\}^3 - 2.293775\times10^{-3}$$



Iteration #1



$$R_{-1}=14000,\ R_0=15000$$

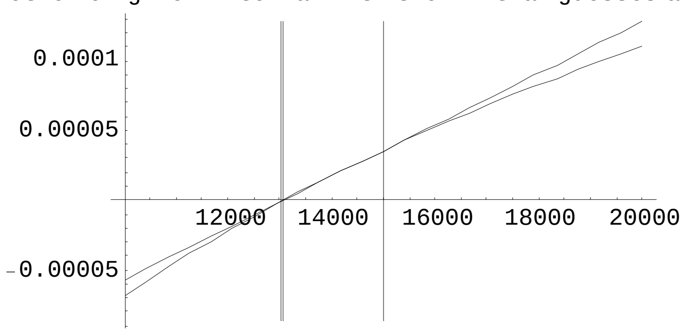
$$R_1=R_0-\frac{f(R_0)(R_0-R_{-1})}{f(R_0)-f(R_{-1})}$$

$$R_1=15000-\frac{(3.5383\times10^{-5})(15000-14000)}{(3.5383\times10^{-5})-(1.7564\times10^{-5})}$$

$$=13014$$

$$|\in_a|=15.258\%$$

Iteration #2



$$R_0 = 15000, R_1 = 13014$$

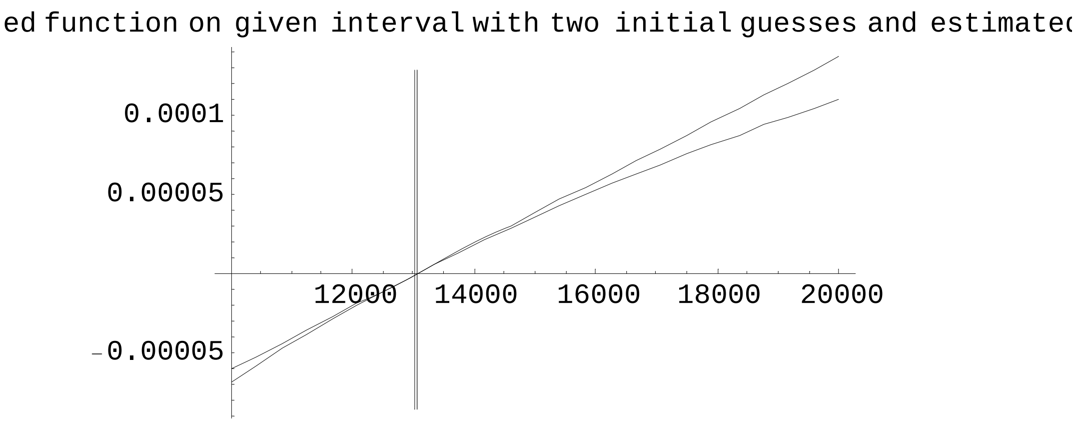
$$R_2 = R_1 - \frac{f(R_1)(R_1 - R_0)}{f(R_1) - f(R_0)}$$

$$R_2 = 13014 - \frac{-1.2659 \times 10^{-6}(13014 - 15000)}{(-1.2659 \times 10^{-6}) - (3.5383 \times 10^{-5})}$$

$$= 13083$$

$$|\in_a| = .52423\%$$

Iteration #3



$$R_1 = 13014, \; R_2 = 13083$$

$$R_3 = R_2 - \frac{f(R_2)(R_2 - R_1)}{f(R_2) - f(R_1)}$$

$$R_3 = 13083 - \frac{(8.8911 \times 10^{-8})(13083 - 13014)}{(8.8911 \times 10^{-8}) - (-1.2659 \times 10^{-6})}$$

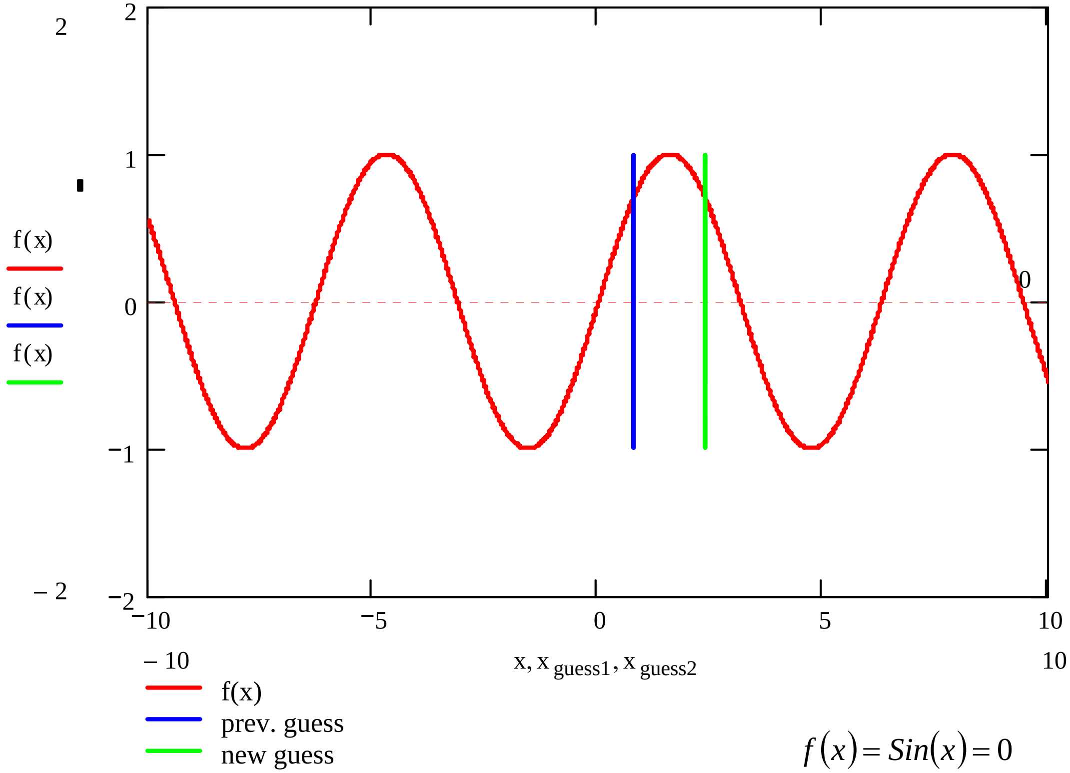
$$= 13078$$

$$|\in_a| = .035026\,\%$$

Advantages

- Converges fast, if it converges
- Requires two guesses that do not need to bracket the root

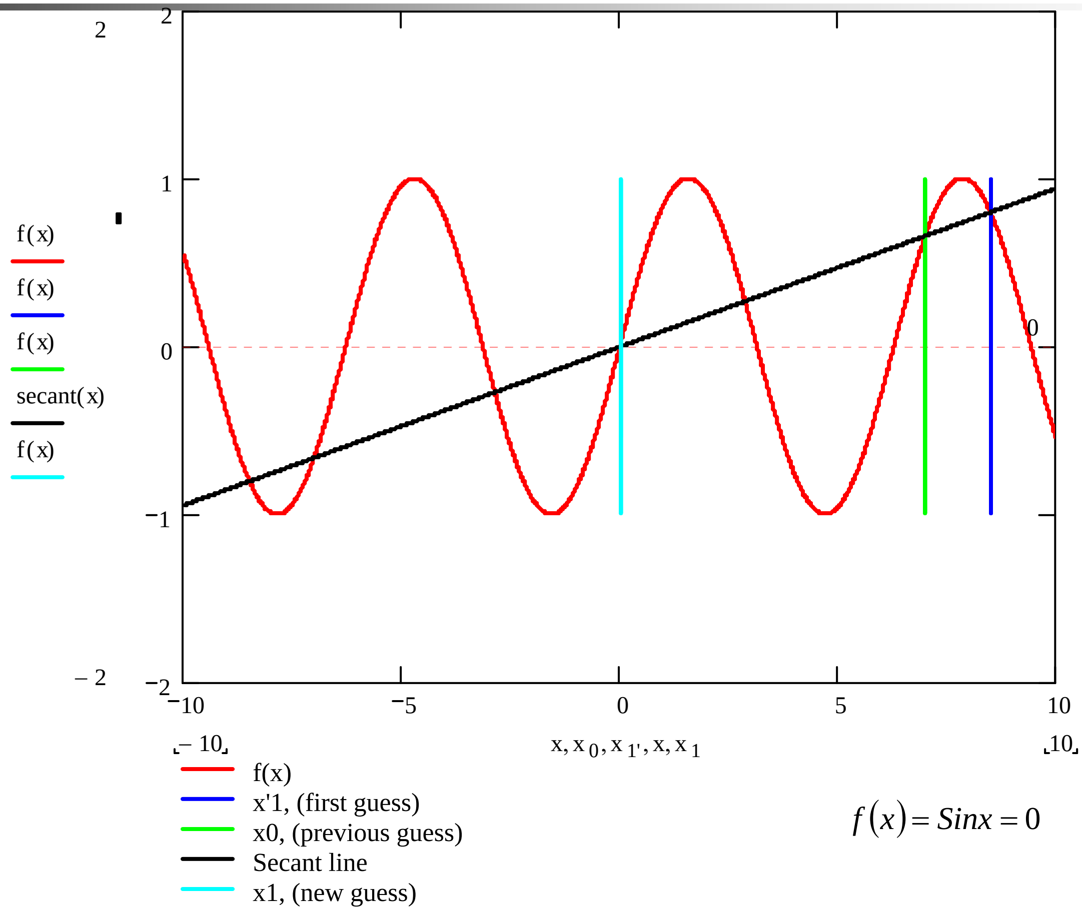
Drawbacks



$f(x) = Sin(x) = 0$

Division by zero

Drawbacks (continued)



$f(x) = Sinx = 0$

Root Jumping