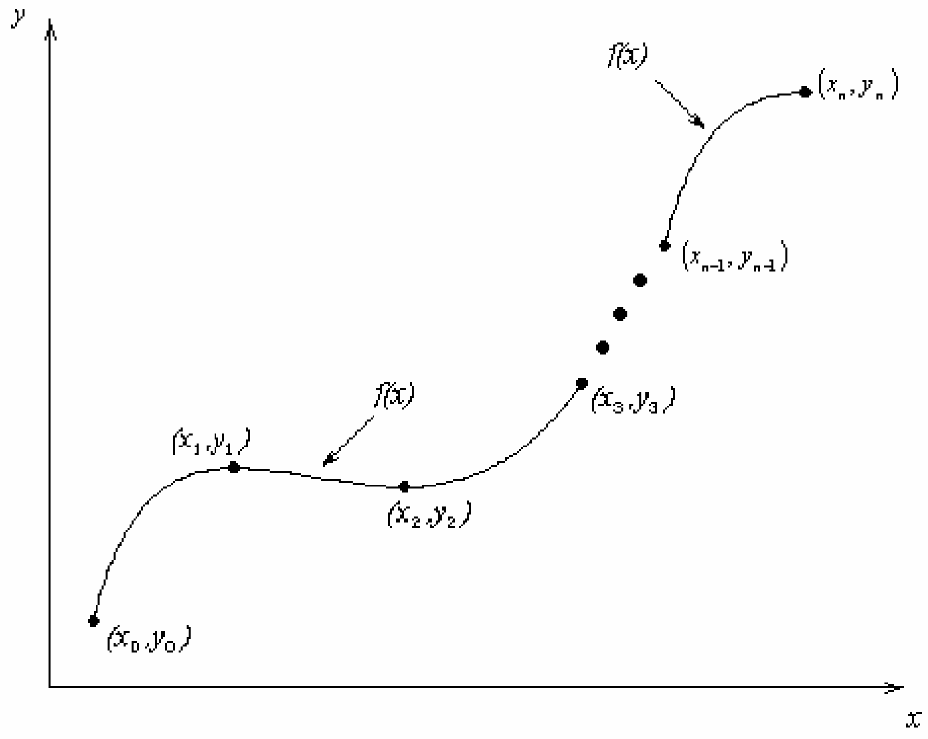
Interpolation

Topic: Direct Method

Major: Electrical

What is Interpolation ?

Given (x_0,y_0), (x_1,y_1), …… (x_n,y_n), find the value of 'y' at a value of 'x' that is not given.



Interpolants

Polynomials are the most common choice of interpolants because they are easy to:

- Evaluate
- Differentiate, and
- Integrate.

Direct Method

Given 'n+1' data points (x_0,y_0), (x_1,y_1),………….. (x_n,y_n), pass a polynomial of order 'n' through the data as given below:

$$y = a_0 + a_1 x + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + a_n x^n .$$

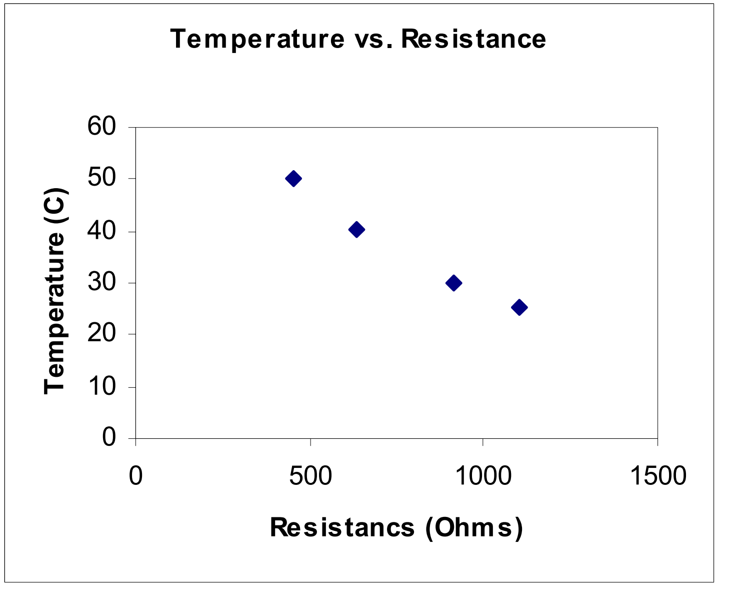
where a_0, a_1,………………. a_n are real constants.

- Set up 'n+1' equations to find 'n+1' constants.
- To find the value 'y' at a given value of 'x', simply substitute the value of 'x' in the above polynomial.

Example

A manufacturer of thermistors makes the following observations on a thermistor. Determine the temperature corresponding to 754.8 ohms using the direct method

R	T
Ohm	$°C$
1101.0	25.113
911.3	30.131
636.0	40.120
451.1	50.128

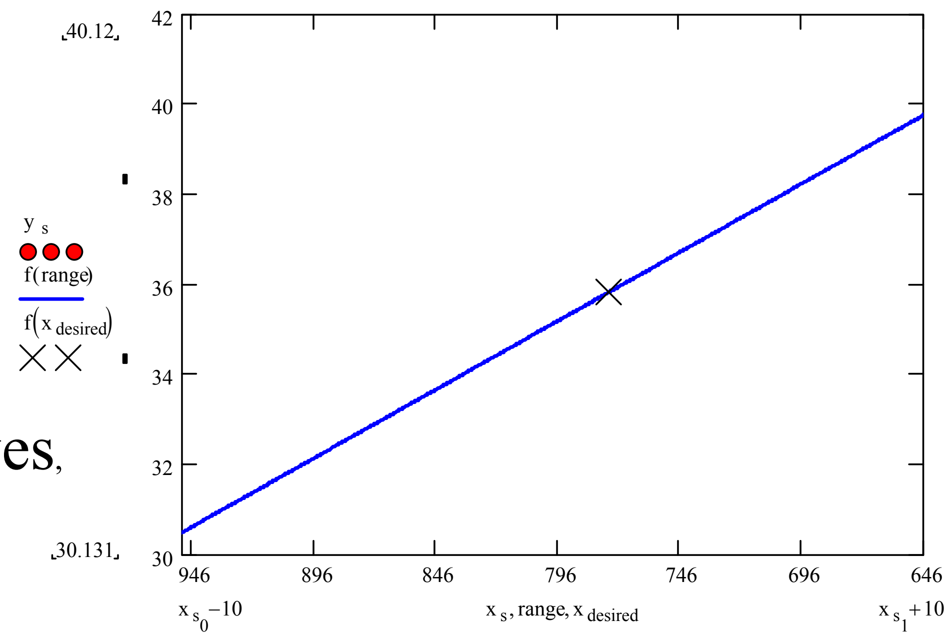


Linear Interpolation

$$T(R) = a_0 + a_1 R$$

$$T(911.3) = a_0 + a_1(911.3) = 30.131$$

$$T(636.0) = a_0 + a_1(636.0) = 40.120$$



Solving the above two equations gives,

$$a_0 = 63.197 \qquad a_1 = -0.03628$$

Hence

$$T(R) = 63.197 - 0.03628R, \;\; 636.0 \le R \le 911.3.$$

$$T(754.8) = 63.197 - 0.03628(754.8) \; = 35.809°C$$

Quadratic Interpolation

$$T(R) = a_0 + a_1 R + a_2 R^2$$

$$T(911.3) = a_0 + a_1(911.3) + a_2(911.3)^2 = 30.131$$

$$T(636.0) = a_0 + a_1(636.0) + a_2(636.0)^2 = 40.120$$

$$T(451.1) = a_0 + a_1(451.1) + a_2(451.1)^2 = 50.128$$

Solving the above three equations gives

$$a_0 = 85.668 \qquad a_1 = -0.09627 \qquad a_2 = 3.8771 * 10^{-5}$$

Quadratic Interpolation (contd)

$$T(R) = 85.668 - 0.09627R + 3.8771*10^{-5}R^2, \quad 451.1 \le R \le 911.3$$

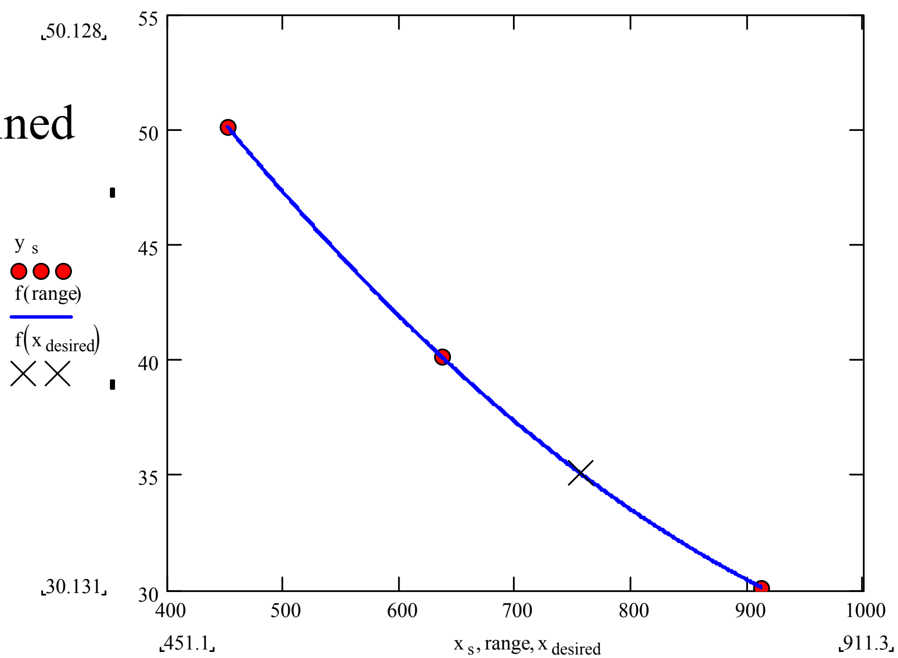
$$T(754.8) = 85.668 - 0.09627(754.8) + 3.8771*10^{-5}(754.8)^2$$

$$= 35.089°C$$

The absolute relative approximate error $|\in_a|$ obtained between the first and second order polynomial is

$$|\in_a| = \left|\frac{35.089 - 35.809}{35.089}\right| \times 100$$

$$= 2.0544\%$$



Cubic Interpolation

$$T(R) = a_0 + a_1 R + a_2 R^2 + a_3 R^3$$

$$T(1101.0) = 25.113 = a_0 + a_1(1101.0) + a_2(1101.0)^2 + a_3(1101.0)^3$$

$$T(911.3) = 30.131 = a_0 + a_1(911.3) + a_2(911.3)^2 + a_3(911.3)^3$$

$$T(636.0) = 40.120 = a_0 + a_1(636.0) + a_2(636.0)^2 + a_3(636.0)^3$$

$$T(451.1) = 50.128 = a_0 + a_1(451.1) + a_2(451.1)^2 + a_3(451.1)^3$$

$a_0 = 92.759$ $a_1 = -0.13093$ $a_2 = 9.2975 * 10^{-5}$ $a_3 = -2.7124 * 10^{-8}$

Cubic Interpolation (contd)

$$T(R) = 92.759 - 0.13093R + 9.2975*10^{-5}R^2 - 2.7124*10^{-8}R^3, \quad 451.1 \le R \le 1101.0$$

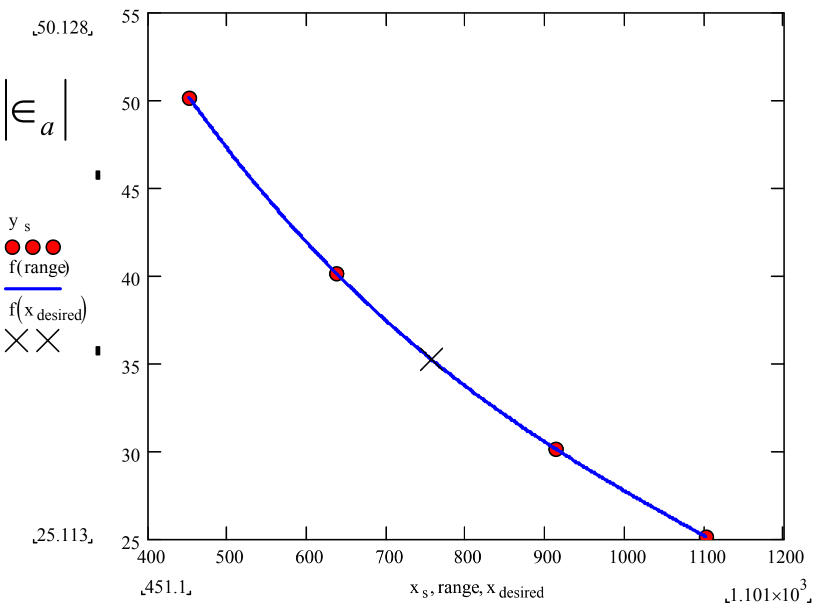
$$T(754.8) = 92.759 - 0.13093(754.8) + 9.2975*10^{-5}(754.8)^2 - 2.7124*10^{-8}(754.8)^3$$

$$= 35.242°C$$

The absolute percentage relative approximate error, $|\in_a|$ between second and third order polynomial is

$$|\in_a| = \left|\frac{35.242 - 35.089}{35.242}\right| \times 100$$

$$= 0.43458\%$$



Comparison Table

Order of Polynomial	1	2	3
Temperature $^\circ C$	35.809	35.089	35.242
Absolute Relative Approximate Error	----------	2.0544%	0.43458%

Actual Calibration

The actual calibration curve used is

$$\frac{1}{T} = a_0 + a_1[\ln R] + a_2[\ln R]^2 + a_3[\ln R]^3$$

substituting $y = \frac{1}{T}$, and $x = \ln R$, gives the calibration curve as

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

Find the calibration curve and find the temperature corresponding to 754.8 ohms. What is the difference between the results from cubic interpolation? In which method is the difference larger, if the actual measured value at 754.8 ohms is 35.285 $°C$?

R	T	x	y
Ohm	$°C$	ln R	$\frac{1}{T}$
1101.0	25.113	7.0040	0.039820
911.3	30.131	6.8149	0.033188
636.0	40.120	6.4552	0.024925
451.1	50.128	6.1117	0.019949

Actual Calibration

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$y(7.0040) = 0.039820 = a_0 + a_1(7.0040) + a_2(7.0040)^2 + a_3(7.0040)^3$$

$$y(6.8149) = 0.033188 = a_0 + a_1(6.8149) + a_2(6.8149)^2 + a_3(6.8149)^3$$

$$y(6.4552) = 0.024925 = a_0 + a_1(6.4552) + a_2(6.4552)^2 + a_3(6.4552)^3$$

$$y(6.1117) = 0.019949 = a_0 + a_1(6.1117) + a_2(6.1117)^2 + a_3(6.1117)^3$$

$a_0 = -2.5964$ $a_1 = 1.2605$ $a_2 = -0.20448$ $a_3 = 0.011173$

Actual Calibration

$$y(x) = -2.5964 + 1.2605x - 0.20448x^2 + 0.011173x^3 \quad 6.1117 \le x \le 7.0040$$

However, since $y = \frac{1}{T}$, and $x = \ln R$,

$$\frac{1}{T} = -2.5964 + 1.2605(\ln R) - 0.20448(\ln R)^2 + 0.011173(\ln R)^3 \quad 451.1 \le R \le 1101.0$$

$$T(R) = \frac{1}{-2.5964 + 1.2605(\ln R) - 0.20448(\ln R)^2 + 0.011173(\ln R)^3} \quad 451.1 \le R \le 1101.0$$

At R = 754.8,

$$T(754.8) = \frac{1}{-2.5964 + 1.2605(\ln 754.8) - 0.20448(\ln 754.8)^2 + 0.011173(\ln 754.8)^3}$$

$$= 35.059°C$$

Actual Calibration

Since the actual measured value at 754.8 ohms is $35.285\,°C$, the absolute relative true error between the value used for Cubic Interpolation is

$$|\in_t| = \left|\frac{35.285 - 35.242}{35.285}\right| \times 100$$

$$= 0.12186\%$$

and for actual calibration is

$$|\in_t| = \left|\frac{35.285 - 35.059}{35.285}\right| \times 100$$

$$= 0.64050\%$$

Therefore, the direct method of cubic polynomial interpolation obtained more accurate results than the actual calibration curve.