Chapter 07.00A

Physical Problem for Integration General Engineering

Problem

A rocket is going vertically up and expels fuel at a velocity $2000\,\text{m/s}$ at a consumption rate of $2100\,\text{kg/s}$. The initial mass of the rocket is $140{,}000\,\text{kg}$. If the rocket starts from rest at $t=0$ seconds, how can I calculate the vertical distance covered by the rocket from $t=8$ to $t=30$ seconds?

Figure 1 A rocket launched into space

Solution

If

$m_0 =$ initial mass of rocket at $t=0$ (kg)

$q =$ rate at which fuel is expelled (kg/sec)

$u =$ velocity at which the fuel is being expelled (m/s)

Then since fuel is expelled from the rocket, the mass of the rocket keeps decreasing with time. The mass of rocket, m at any time t

$$m = m_0 - qt$$

The forces on the rocket at any time are found by applying Newton's second law of motion. Then

$$\sum F = ma$$

$$uq - mg = ma$$
$$uq - (m_0 - qt)g = (m_0 - qt)a$$

where

$$g = \text{ acceleration due to gravity } (\text{m/s}^2)$$
$$a = \frac{uq}{m_0 - qt} - g$$
$$\frac{d^2 x}{dt^2} = \frac{uq}{m_0 - qt} - g$$
$$\frac{dx}{dt} = -u\log_e(m_0 - qt) - gt + C$$

Since the rocket starts from rest

$$\frac{dx}{dt} = 0 \text{ at } t = 0$$
$$0 = u\log_e(m_0) + C$$
$$C = u\log_e(m_0)$$

Hence

$$\frac{dx}{dt} = -u\log_e(m_0 - qt) - gt + u\log_e(m_0)$$
$$\frac{dx}{dt} = u\log_e\left(\frac{m_0}{m_0 - qt}\right) - gt$$

Then the distance covered by the rocket from $t = t_0$ to $t = t_1$ is,

$$x = \int_{t_0}^{t_1}\left[u\log_e\left(\frac{m_0}{m_0 - qt}\right) - gt\right]dt$$

Let us substitute the values into the above equation. A rocket is going vertically up and expels fuel at a velocity 2000 m/s at a consumption rate of 2100 kg/s. The initial mass of the rocket is $140{,}000$ kg. If the rocket starts from rest at $t = 0$ seconds, how can I calculate the vertical distance covered by the rocket from $t = 8$ to $t = 30$ seconds?

Substituting

$$u = 2000 \text{ m/s}$$
$$m_0 = 140000 \text{ kg}$$
$$q = 2100 \text{ kg/s}$$
$$g = 9.8 \text{ m/s}^2$$
$$t_0 = 8 \text{ s}$$
$$t_1 = 30 \text{ s}$$

$$x = \int_{8}^{30}\left(2000\ln\left[\frac{140000}{140000 - 2100t}\right] - 9.8t\right)dt$$

INTEGRATION	
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