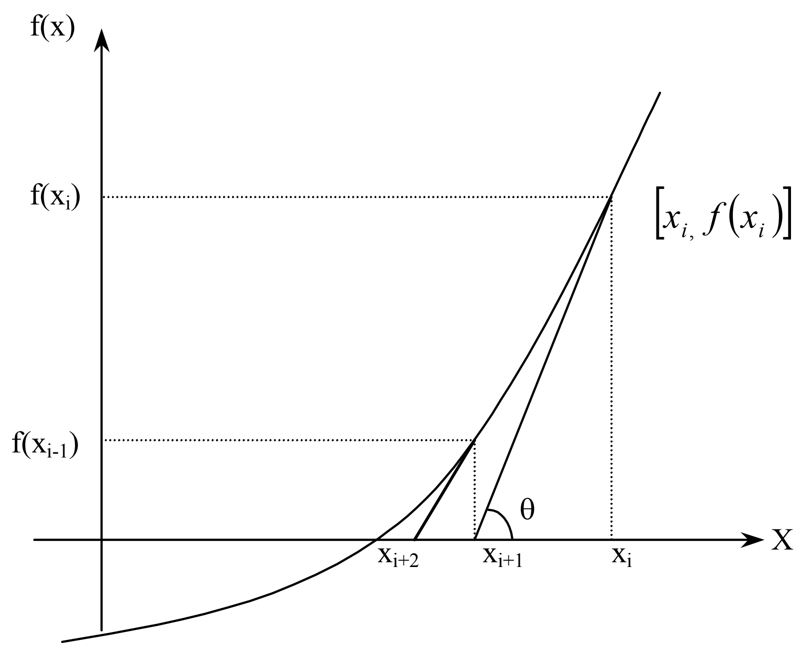
Roots of a Nonlinear Equation

Topic: Newton-Raphson Method

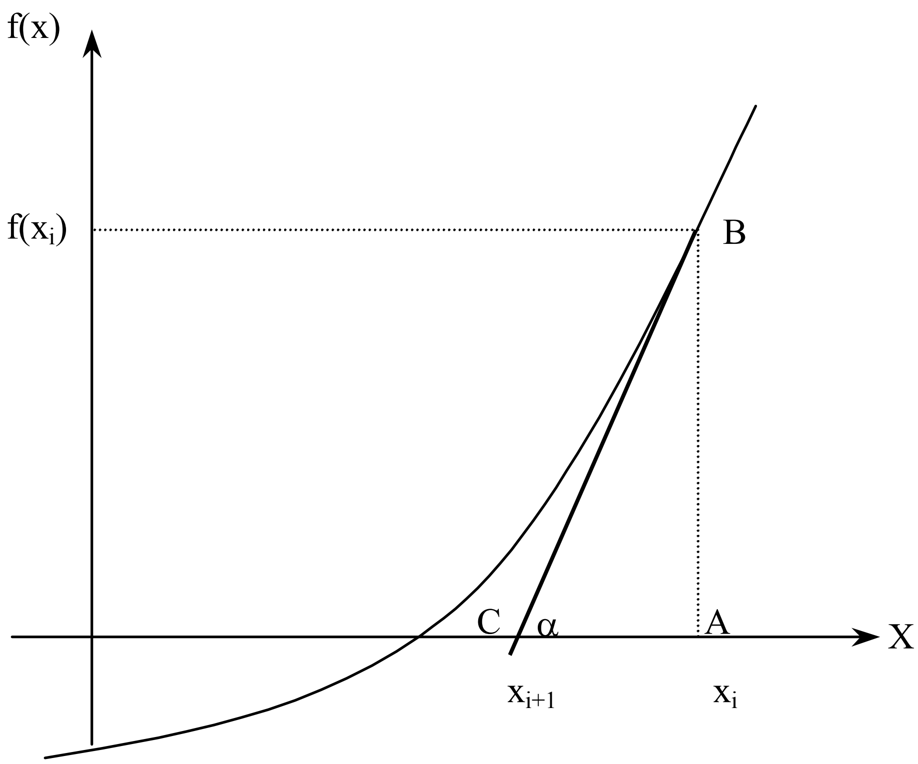
Major: Industrial Engineering

Newton-Raphson Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$



Derivation



$$\tan(\alpha) = \frac{AB}{AC}$$

$$f'(x_i) = \frac{f(x_i)}{x_i - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Algorithm for Newton-Raphson Method

Step 1

Evaluate $f'(x)$ symbolically

Step 2

Calculate the next estimate of the root

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Find the absolute relative approximate error

$$|\in_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \text{ x } 100$$

Step 3

- Find if the absolute relative approximate error is greater than the pre-specified relative error tolerance.

- If so, go back to step 2, else stop the algorithm.

- Also check if the number of iterations has exceeded the maximum number of iterations.

Example

- You are working for a start-up computer assembly company and have been asked to determine the minimum number of computers that the shop will have to sell to make a profit.

The equation that gives the minimum number of computers 'x' to be sold after considering the total costs and the total sales is:

$$f(x) = 40x^{1.5} - 875x + 35000 = 0$$

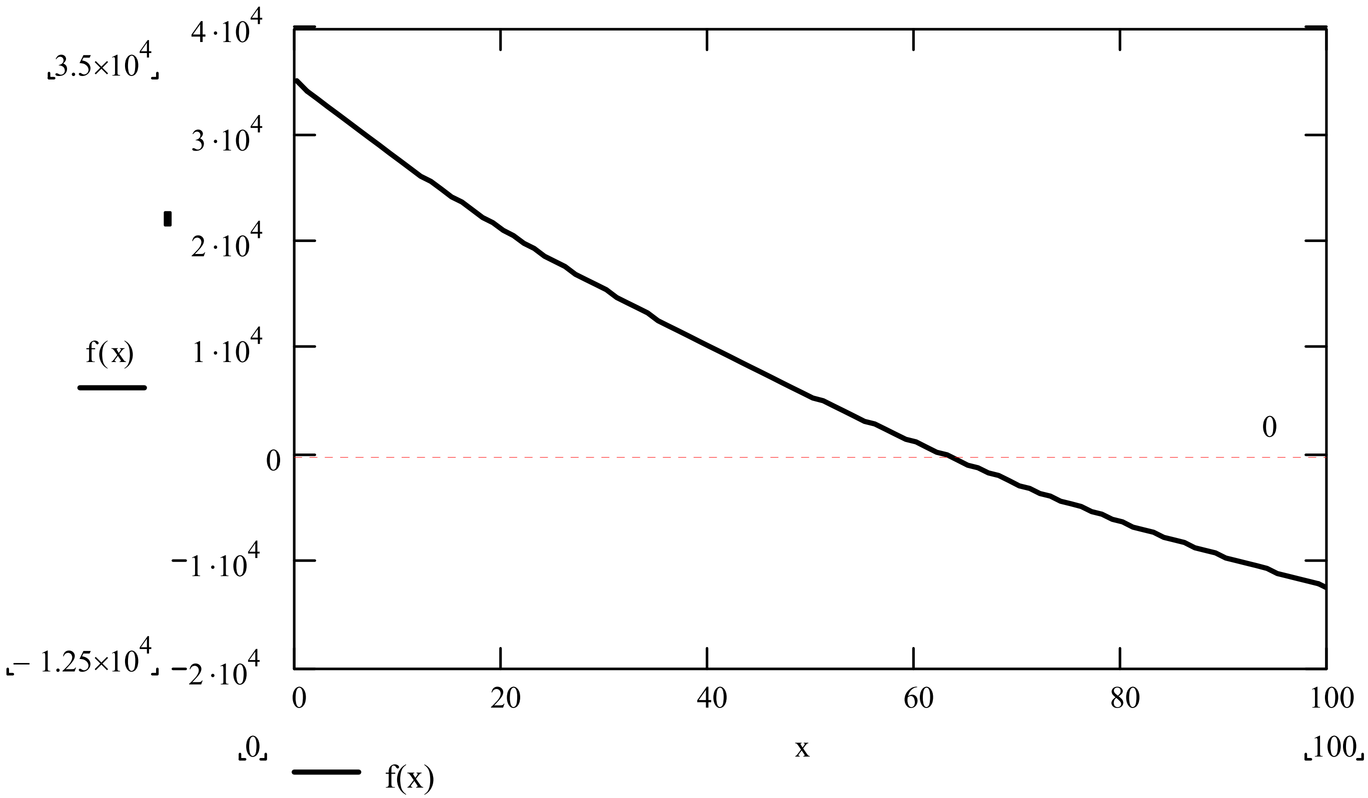
Solution

Use the Newton method of finding roots of equations to find

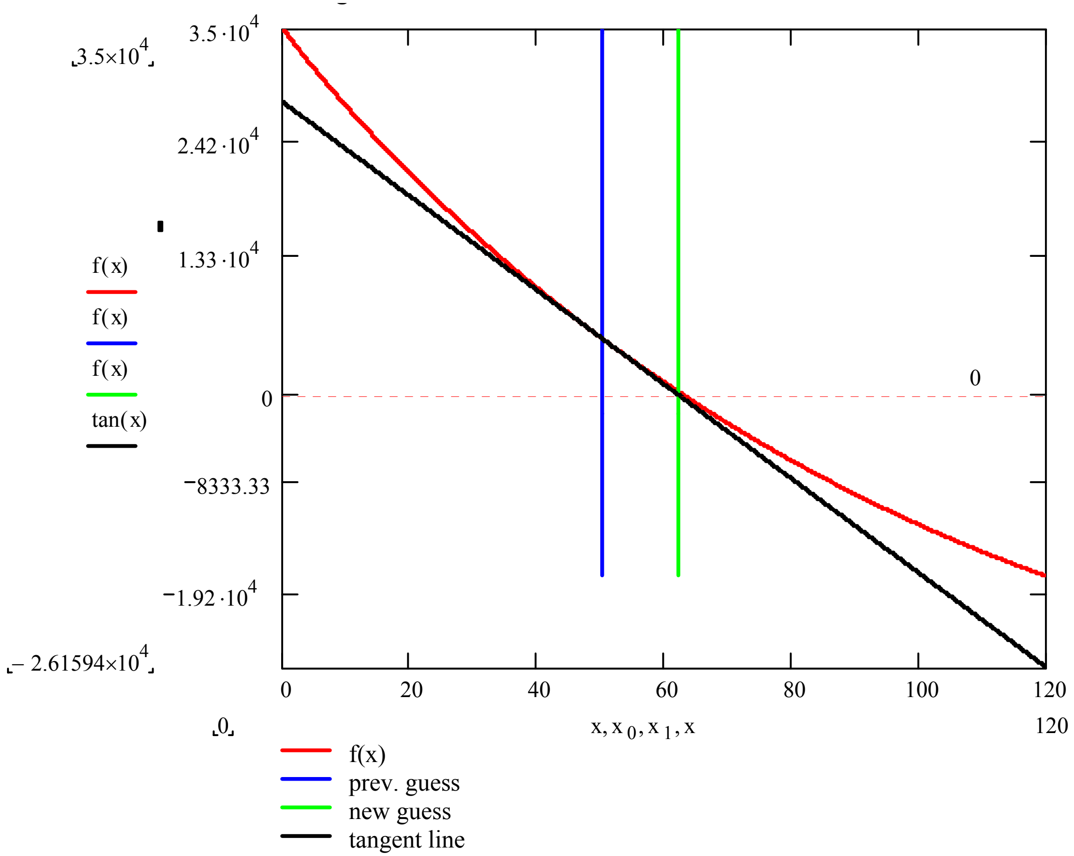
- The minimum number of computers that need to be sold to make a profit. Conduct three iterations to estimate the root of the above equation.
- Find the absolute relative approximate error at the end of each iteration, and
- The number of significant digits at least correct at the end of each iteration.

Graph of function f(x)

$$f(x) = 40x^{1.5} - 875x + 35000 = 0$$



Iteration #1



$$x_0 = 50$$

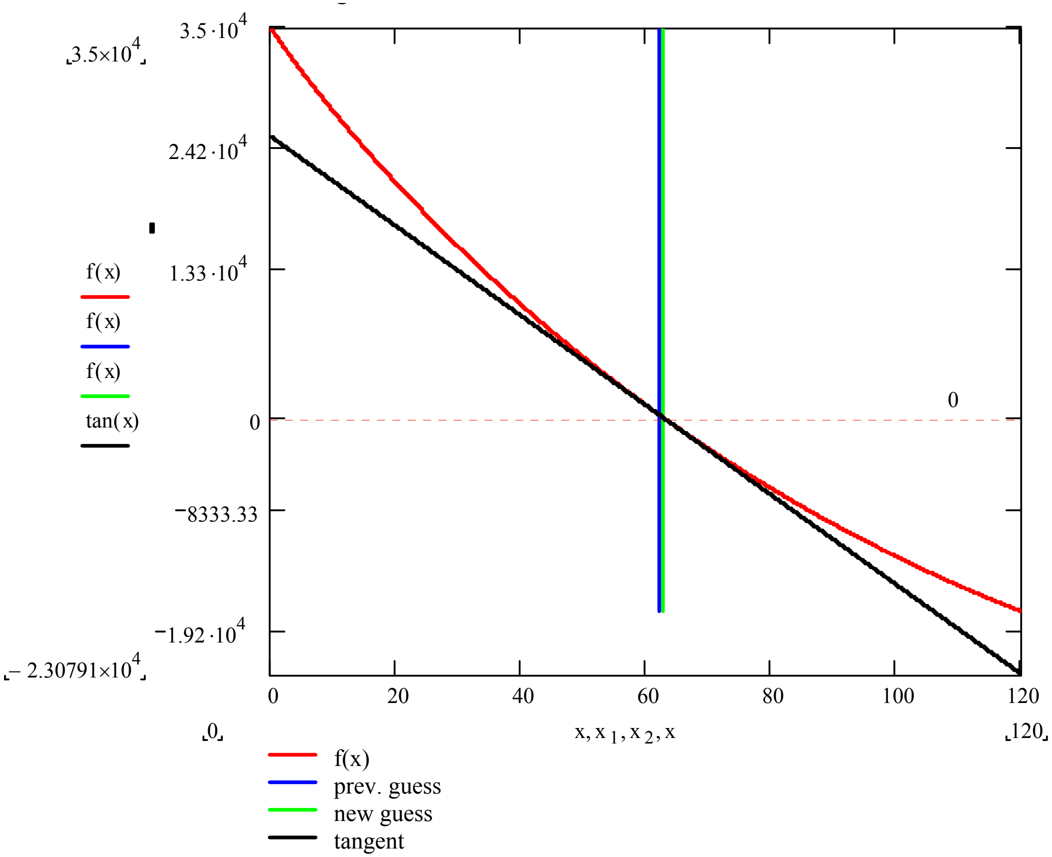
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 50 - \frac{5392.14}{-450.736}$$

$$= 61.963$$

$$|\in_a| = 19.30663\ \%$$

Iteration #2



$$x_1 = 61.963$$

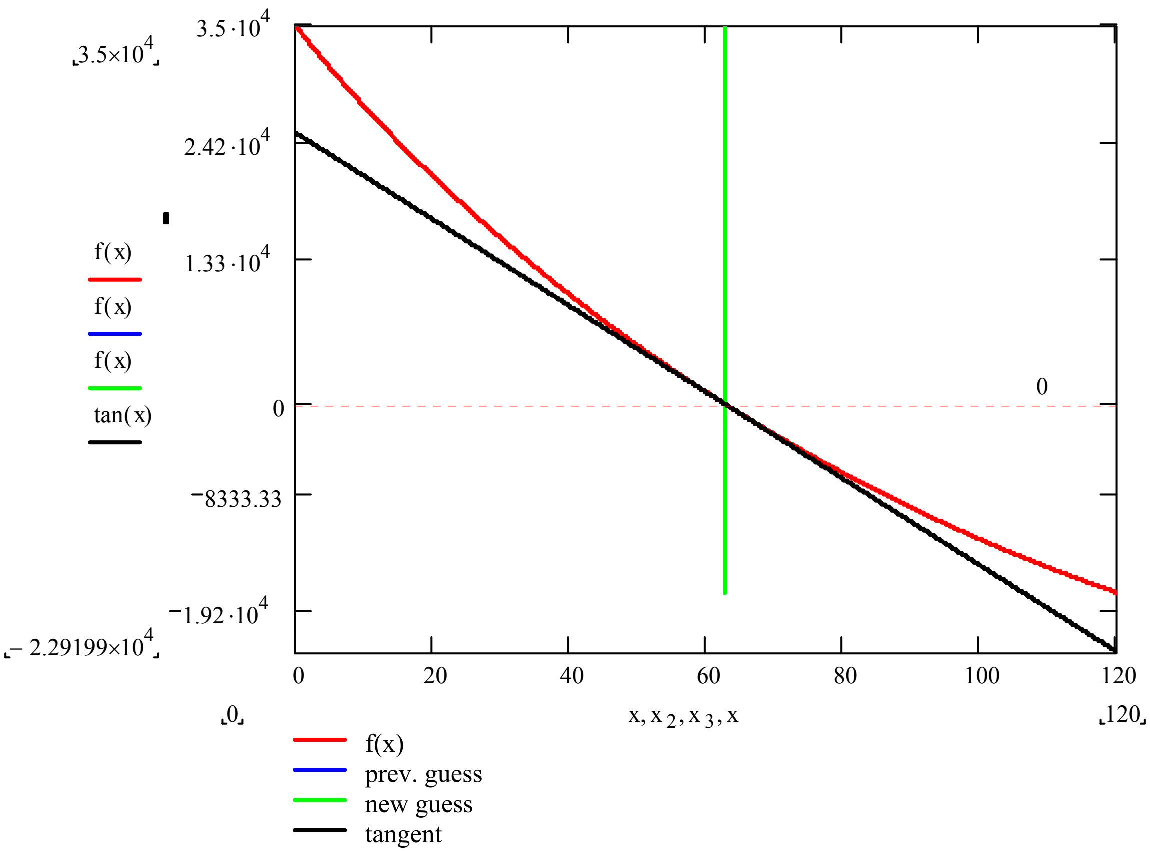
$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 61.963 - \frac{292.4532}{-402.7007}$$

$$= 62.6892$$

$$|\in_a| = 1.1585\ \%$$

Iteration #3



$$x_2 = 62.6892$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 62.6892 - \frac{1.0031}{-399.941}$$

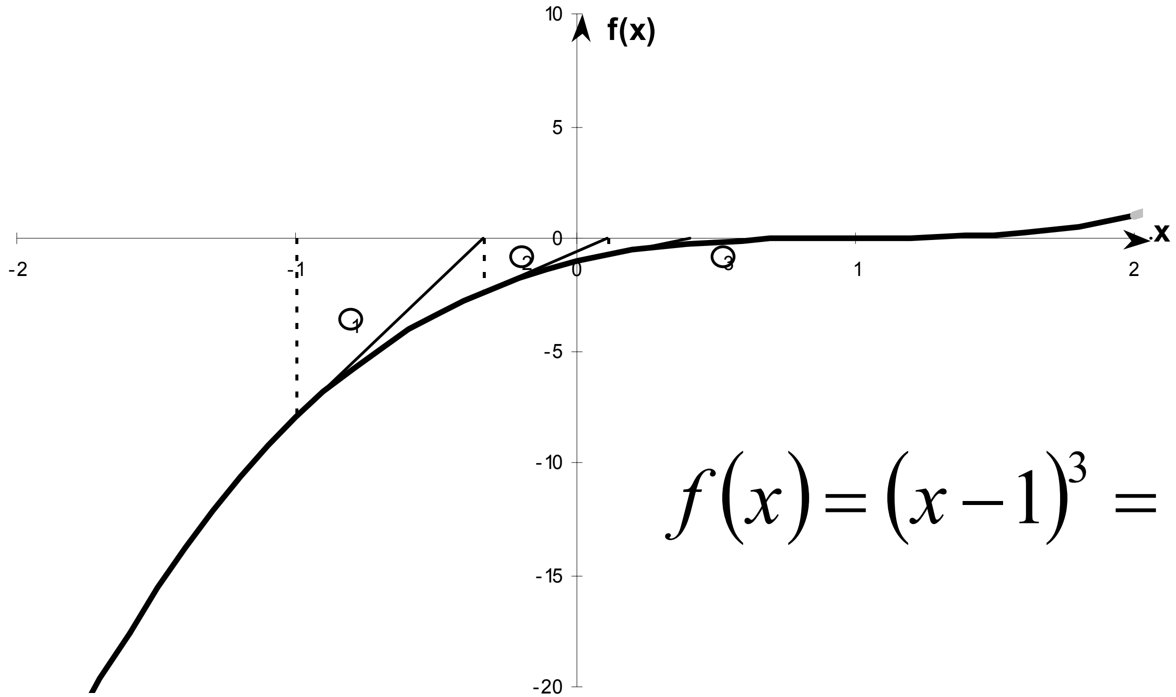
$$= 62.6917$$

$$|\in_a| = 4.006 \times 10^{-5}\%$$

Advantages

- Converges fast, if it converges
- Requires only one guess

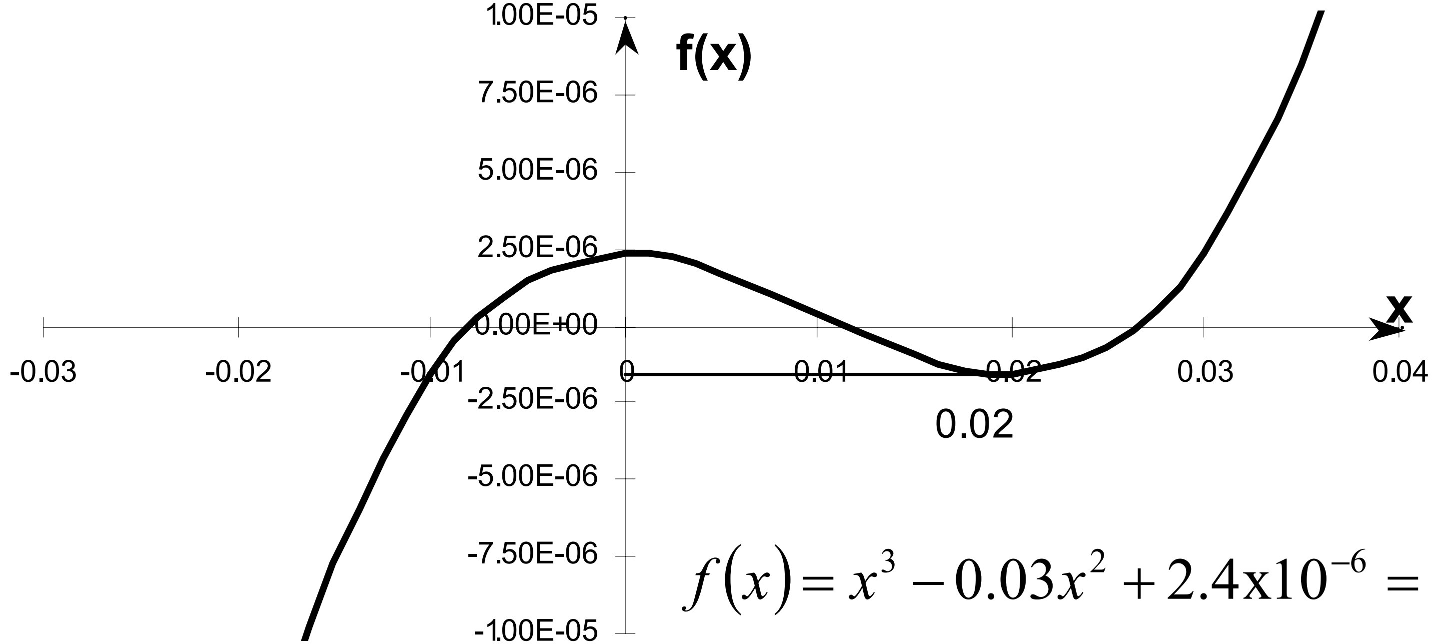
Drawbacks



$$f(x) = (x-1)^3 = 0$$

Inflection Point

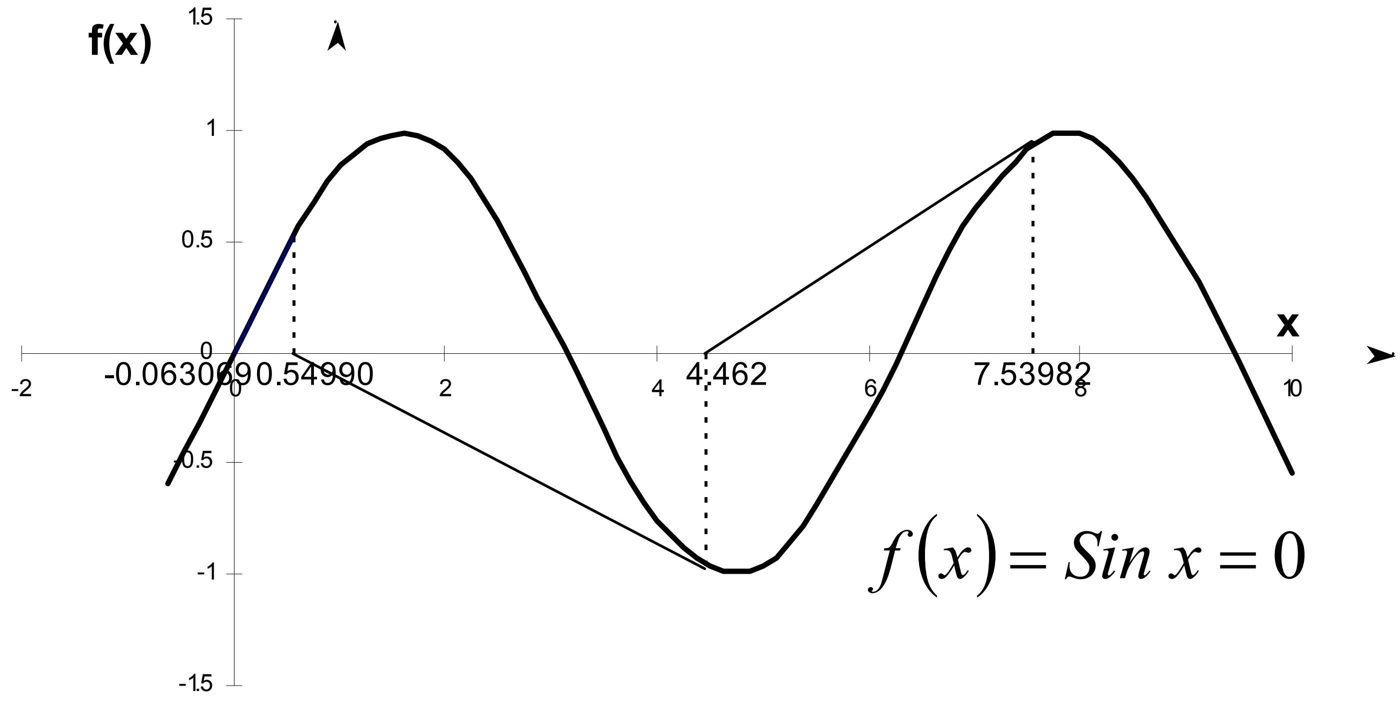
Drawbacks (continued)



$$f(x) = x^3 - 0.03x^2 + 2.4\text{x}10^{-6} = 0$$

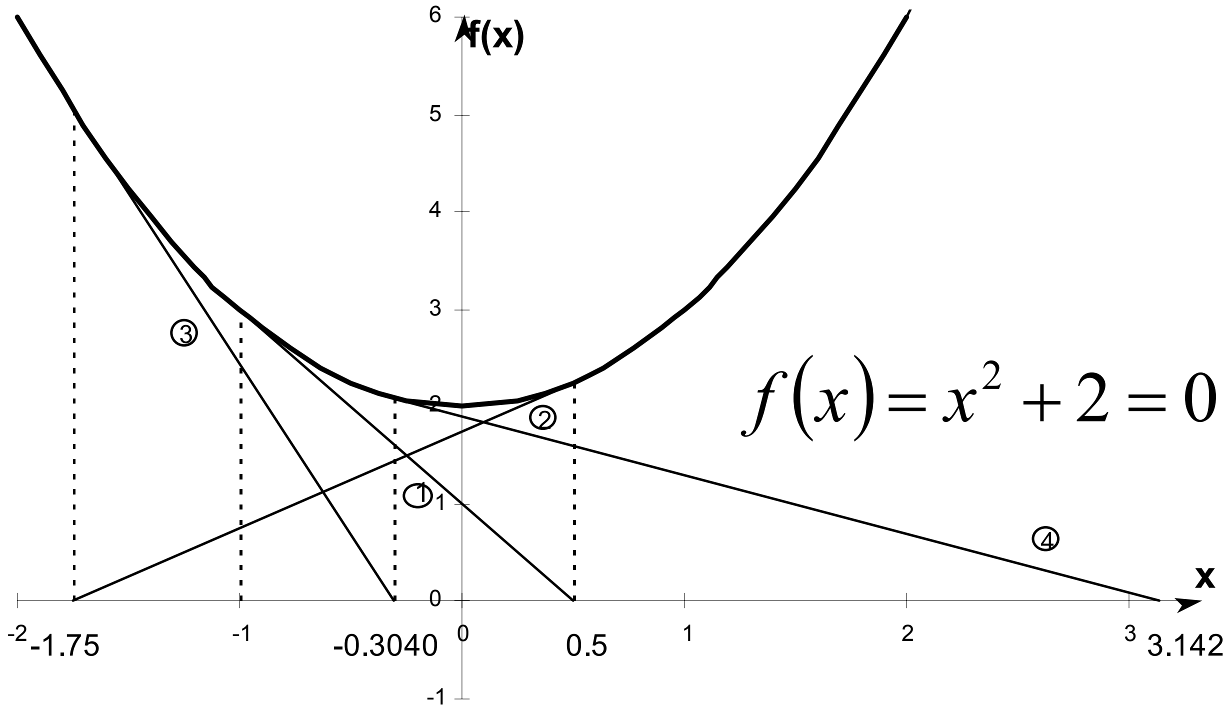
Division by zero

Drawbacks (continued)



Root Jumping

Drawbacks (continued)



$$f(x) = x^2 + 2 = 0$$

Oscillations near Local Maxima or Minima