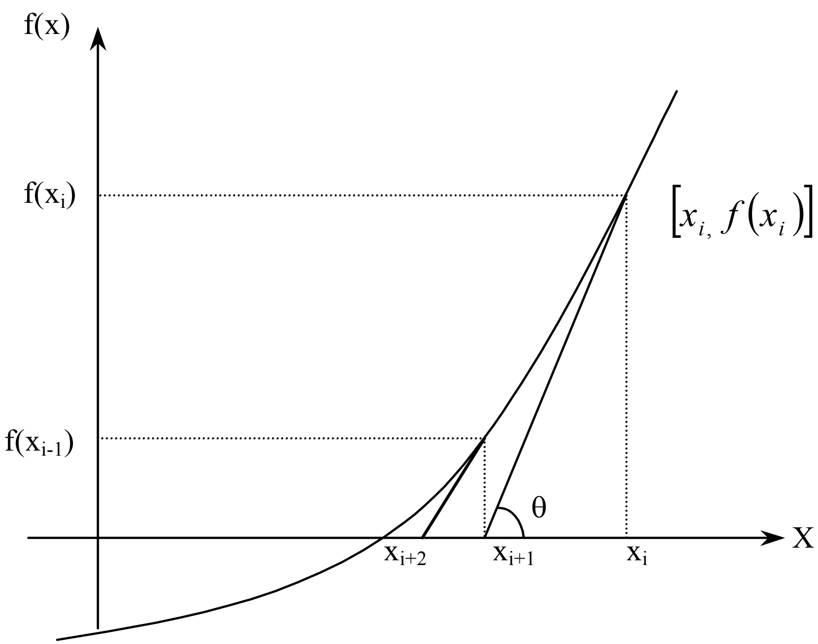
Roots of a Nonlinear Equation

Topic: Secant Method

Major: Industrial Engineering

Secant Method



Newton's Method

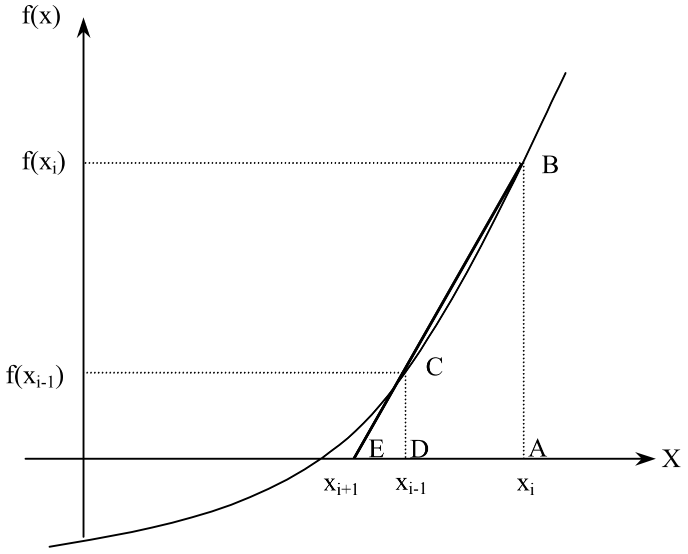
$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Approximate the derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Secant Method



Geometric Similar Triangles

$$\frac{AB}{AE} = \frac{DC}{DE}$$

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Algorithm for Secant Method

Step 1

Calculate the next estimate of the root from two initial guesses

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Find the absolute relative approximate error

$$\left|\in_a\right| = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \text{ x } 100$$

Step 2

Find if the absolute relative approximate error is greater than the prespecified relative error tolerance.

If so, go back to step 1, else stop the algorithm.

Also check if the number of iterations has exceeded the maximum number of iterations.

Example

- You are working for a start-up computer assembly company and have been asked to determine the minimum number of computers that the shop will have to sell to make a profit.

The equation that gives the minimum number of computers 'x' to be sold after considering the total costs and the total sales is:

$$f(x) = 40x^{1.5} - 875x + 35000 = 0$$

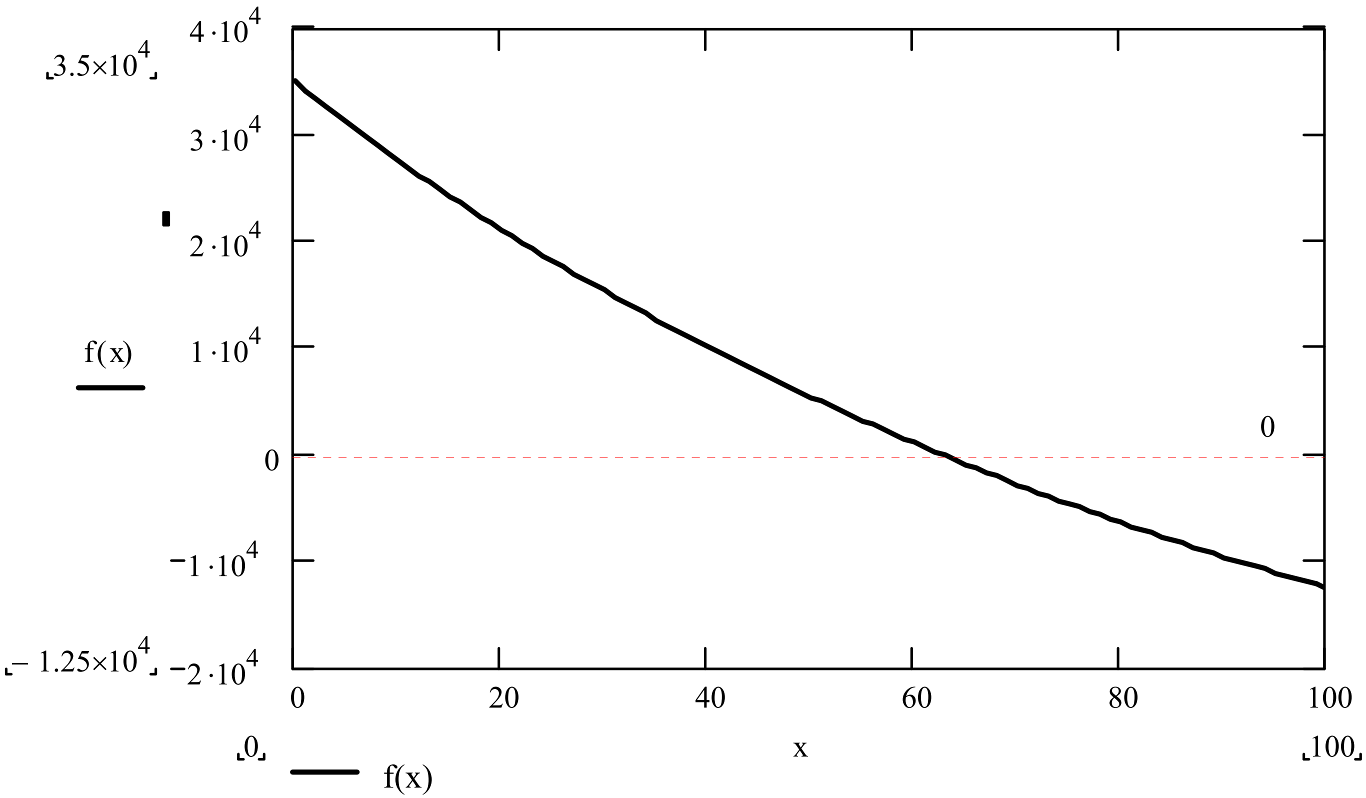
Solution

Use the Secant method of finding roots of equations to find

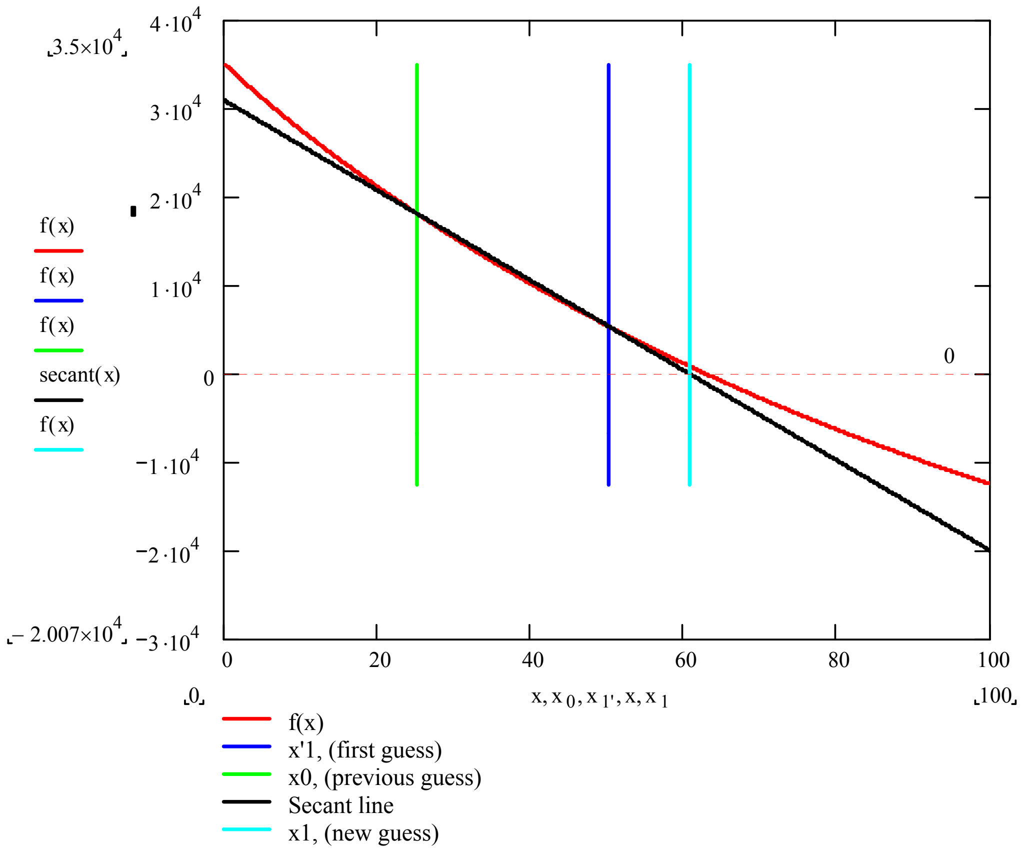
- The minimum number of computers that need to be sold to make a profit. Conduct three iterations to estimate the root of the above equation.
- Find the absolute relative approximate error at the end of each iteration, and
- The number of significant digits at least correct at the end of each iteration.

Graph of function f(x)

$$f(x) = 40x^{1.5} - 875x + 35000 = 0$$



Iteration #1



$$x_{-1} = 25, x_0 = 50$$

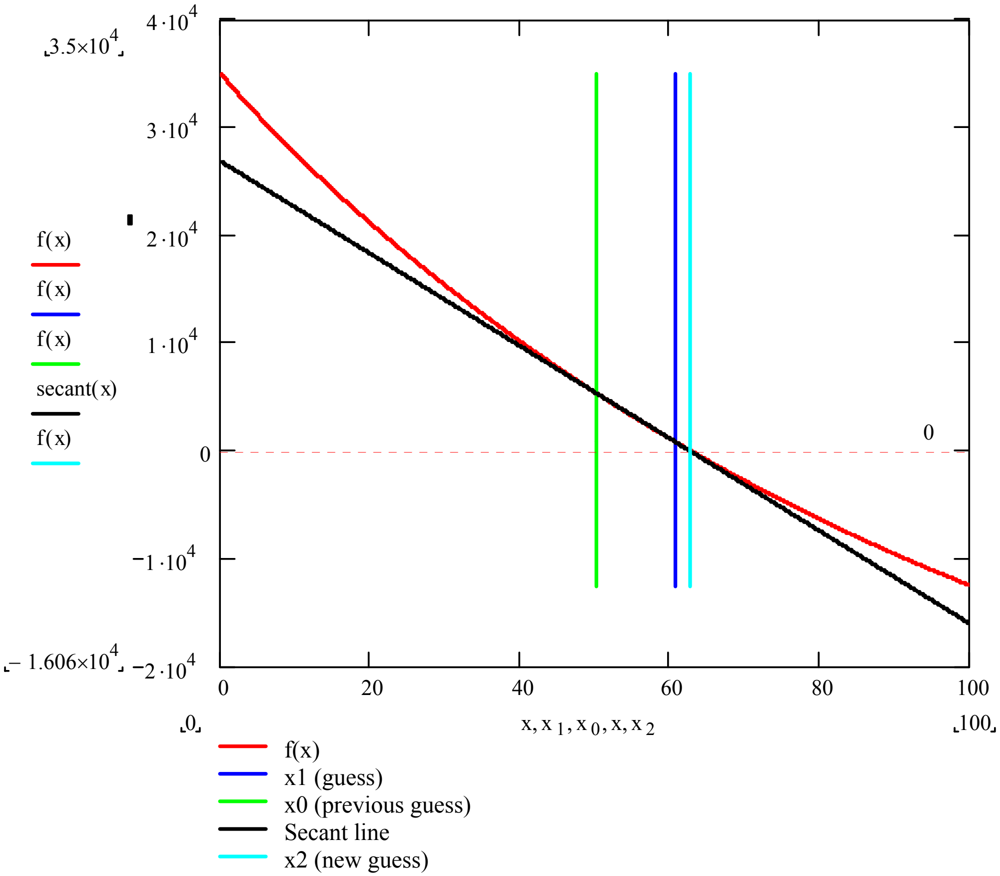
$$x_1 = x_0 - \frac{f(x_0)(x_0 - x_{-1})}{f(x_0) - f(x_{-1})}$$

$$x_1 = 50 - \frac{(5392)(50-25)}{(5392)-(18130)}$$

$$= 60.5871$$

$$|\in_a| = 17.474\%$$

Iteration #2



$$x_0 = 50, x_1 = 60.5871$$

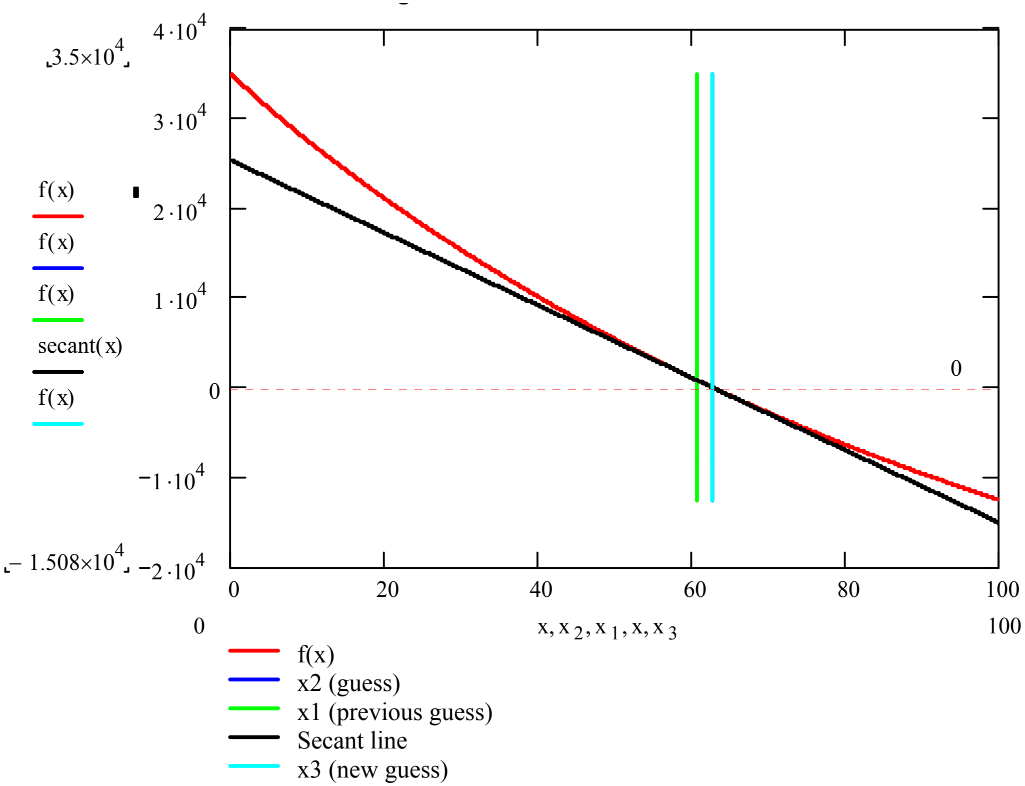
$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = 60.5871 - \frac{850.133(60.5871 - 50)}{(850.133) - (5392)}$$

$$= 62.5687$$

$$|\in_a| = 3.1672\%$$

Iteration #3



$$x_1 = 60.5871, \; x_2 = 62.5687$$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$x_3 = 62.5687 - \frac{(49.219)(62.5687 - 60.5871)}{(49.219) - (850.133)}$$

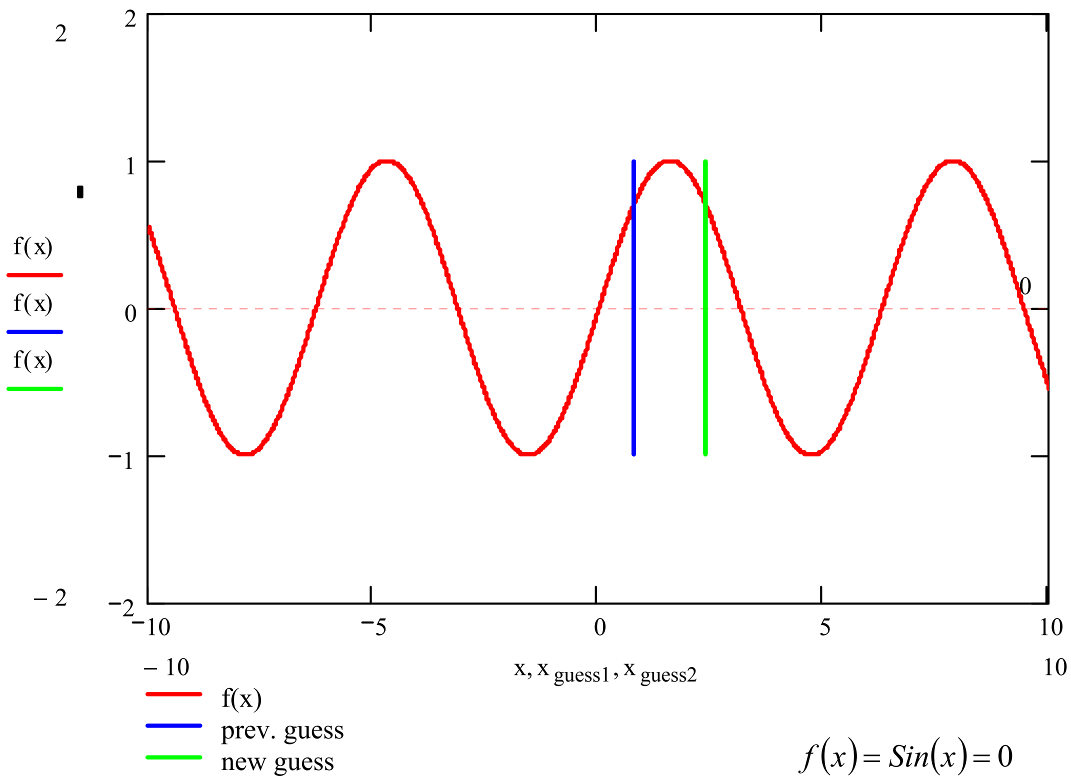
$$= 62.6905$$

$$|\epsilon_a| = 0.1943 \;\%$$

Advantages

- Converges fast, if it converges
- Requires two guesses that do not need to bracket the root

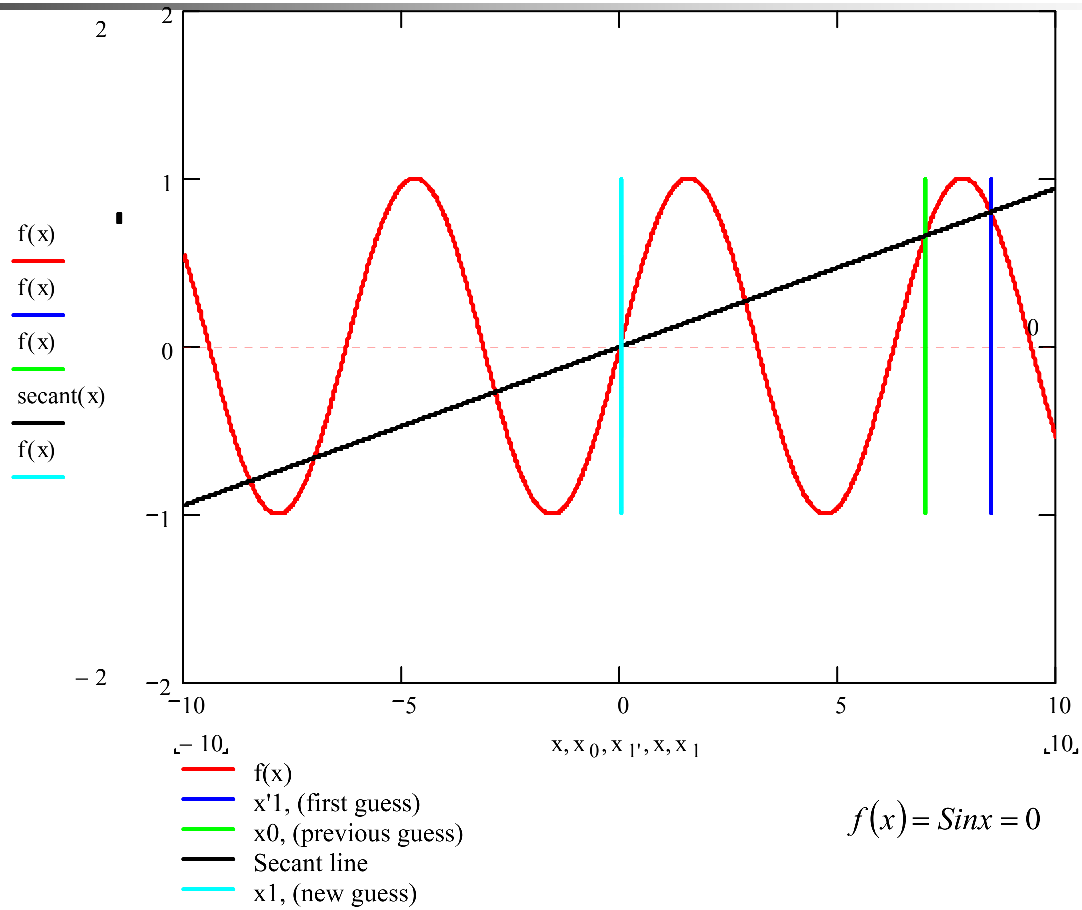
Drawbacks



$f(x) = Sin(x) = 0$

Division by zero

Drawbacks (continued)



$f(x) = Sinx = 0$

Root Jumping