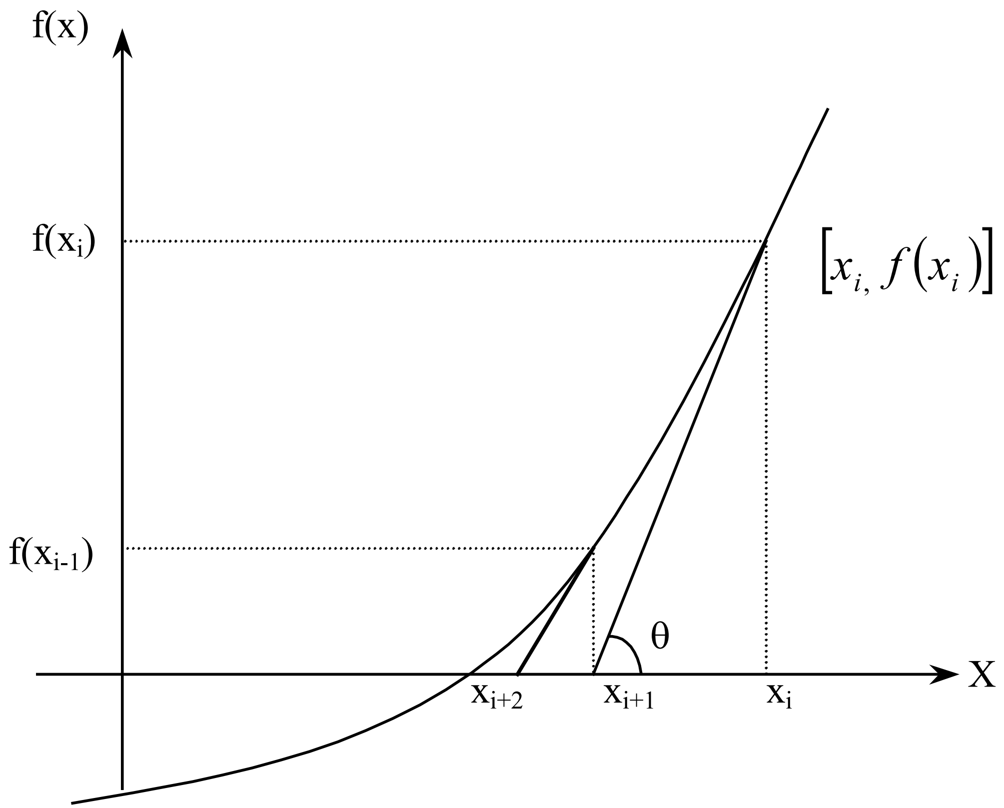
Roots of a Nonlinear Equation

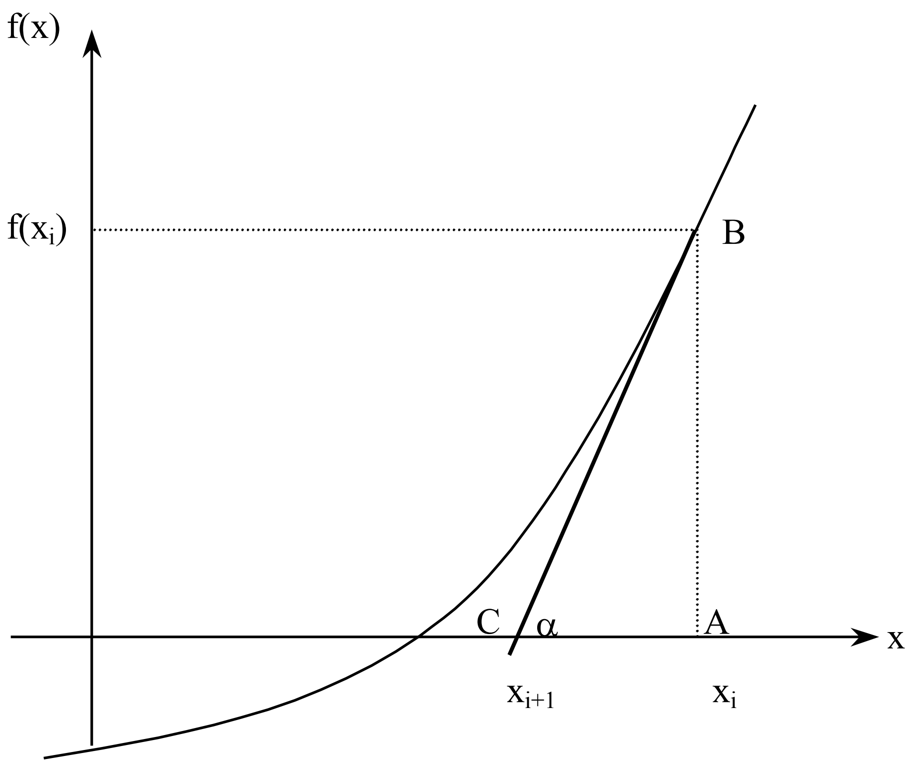
Topic: Newton-Raphson Method

Major: Mechanical Engineering

Newton-Raphson Method



Derivation



$$\tan(\alpha) = \frac{AB}{AC}$$

$$f'(x_i) = \frac{f(x_i)}{x_i - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Algorithm for Newton-Raphson Method

Step 1

Evaluate $f'(x)$ symbolically

Step 2

Calculate the next estimate of the root

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Find the absolute relative approximate error

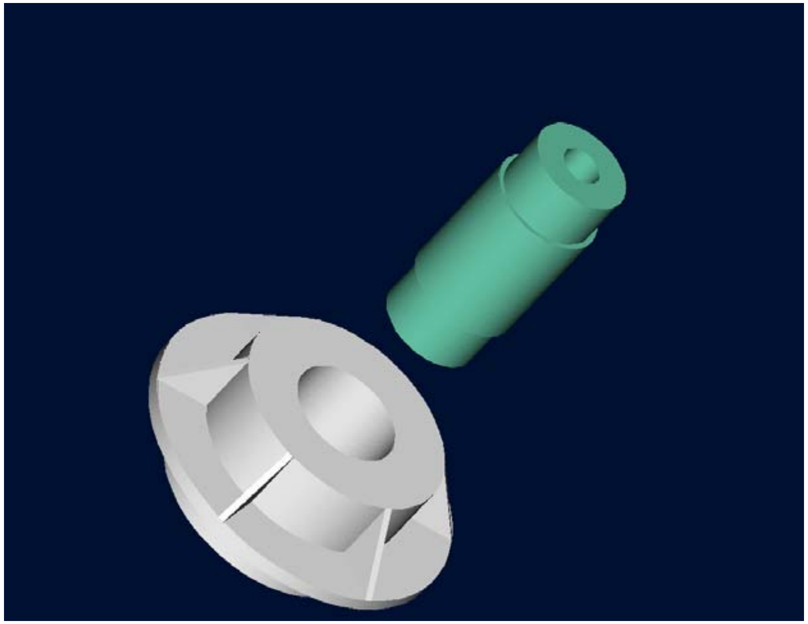
$$|\in_a| = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \text{ x } 100$$

Step 3

- Find if the absolute relative approximate error is greater than the pre-specified relative error tolerance.
- If so, go back to step 2, else stop the algorithm.
- Also check if the number of iterations has exceeded the maximum number of iterations.

Example

- A trunnion has to be cooled before it is shrink fit into a steel hub

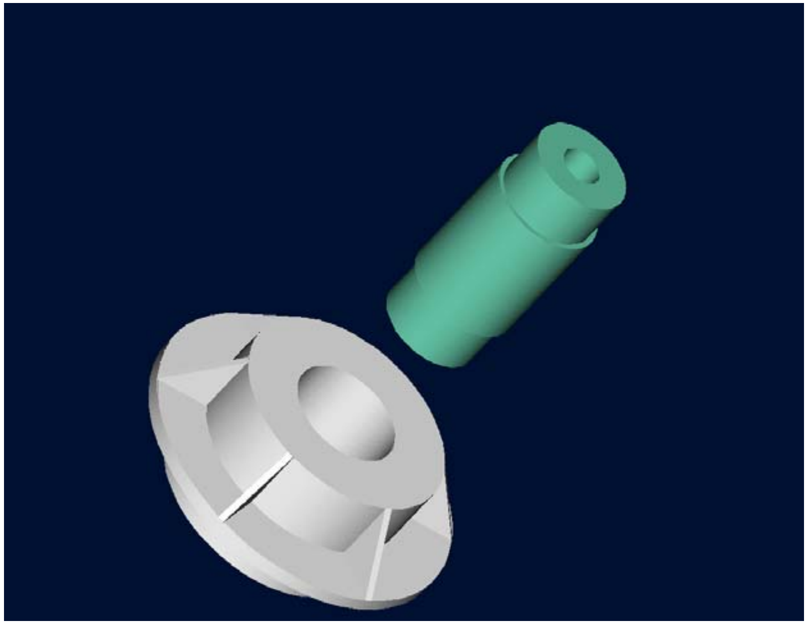


Problem

The equation that gives the temperature 'T' to which it has to be cooled to obtain the desired contraction is given by:

$$f(T) = -0.50598 \times 10^{-10} T^3 + 0.38292 \times 10^{-7} T^2 + 0.74363 \times 10^{-4} T + 0.88318 \times 10^{-2} = 0$$

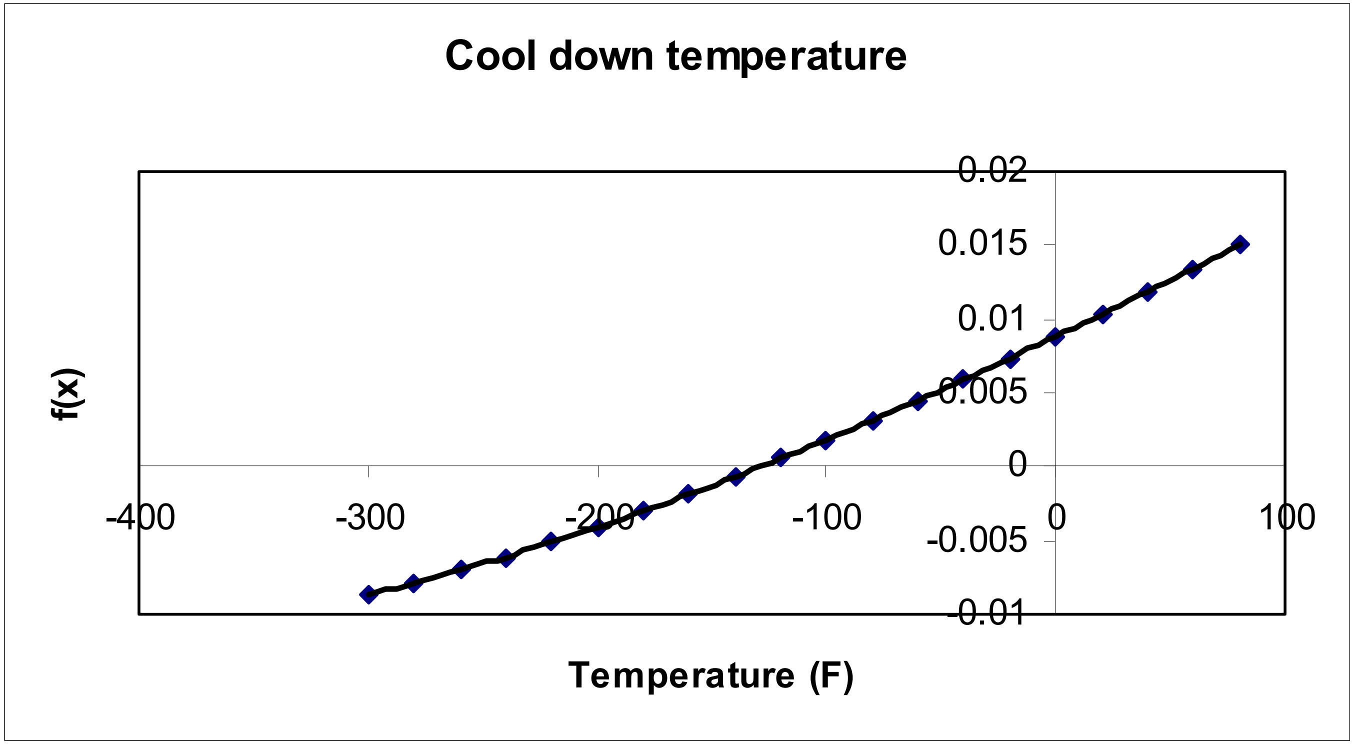
Use the Newton-Raphson method of finding roots of equations to find the temperature to which the trunnion should be cooled down to. Conduct three iterations.



Graph of function f(x)

$$f(T) = -0.50598 \text{ x} 10^{-10} T^3 - 0.38292 \text{ x} 10^{-7} T^2 + 0.74363 \text{ x} 10^{-4} T + 0.88318 \text{ x} 10^{-2} = 0$$

$$f'(T) = -1.51794 \text{ x } 10^{-10} T^2 + 0.76584 \text{ x } 10^{-7} T + 0.74363 \text{ x } 10^{-4}$$



Iteration #1

$$f(T) = -0.50598\text{x}10^{-10}T^3 - 0.38292\text{x}10^{-7}T^2 + 0.74363\text{x}10^{-4}T + 0.88318\text{x}10^{-2} = 0$$

$$f'(T) = -1.51794\text{ x }10^{-10}T^2 + 0.76584\text{ x }10^{-7}T + 0.74363\text{ x }10^{-4}$$

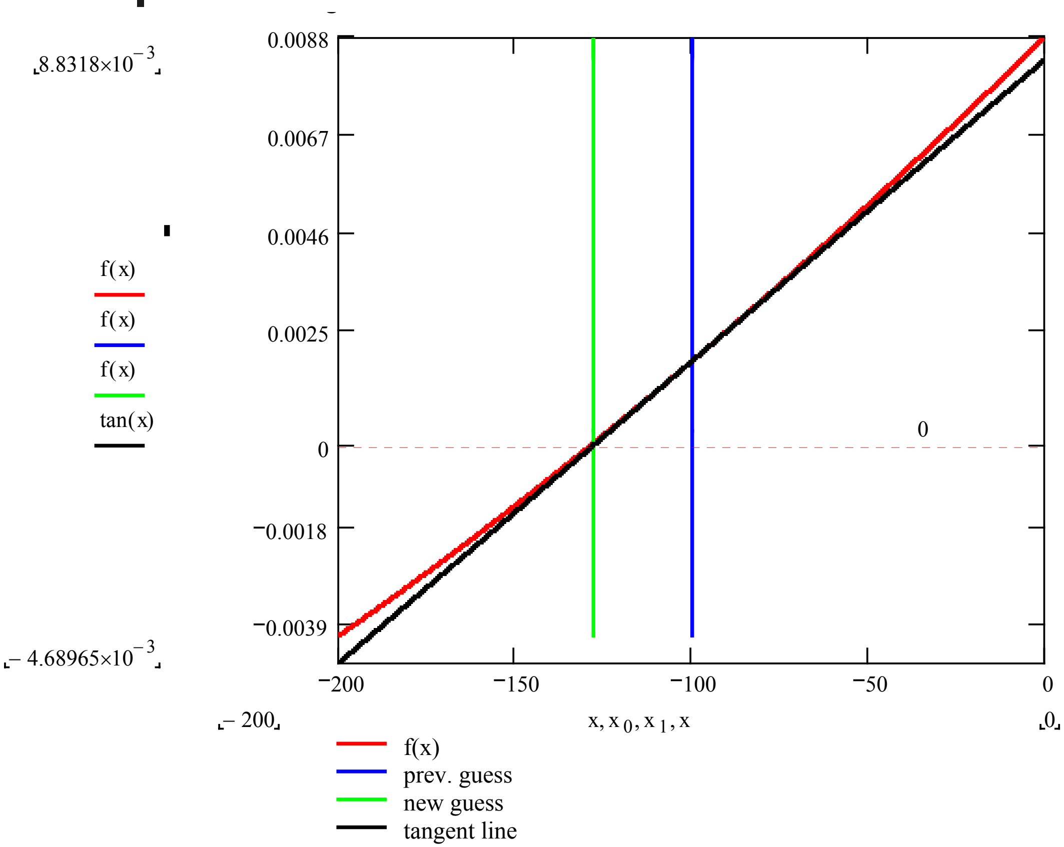
$$T_0 = -100 \qquad T_1 = T_0 - \frac{f(T_0)}{f'(\mathrm{T}_0)}$$

$$f(-100) = -0.50598\text{x}10^{-10}(-100)^3 - 0.38292\text{x}10^{-7}(-100)^2 + 0.74363\text{x}10^{-4}(-100)$$
$$+ 0.88318\text{x}10^{-2} = 1.829\text{x}10^{-3}$$

$$f'(-100) = -1.51794\text{ x }10^{-10}(-100)^2 + 0.76584\text{ x }10^{-7}(-100) + 0.74363\text{ x }10^{-4}$$
$$= 6.5186\text{x}10^{-5}$$

$$T_1 = -100 - \frac{1.829\text{ x}10^{-3}}{6.5186\text{ x}10^{-5}}$$
$$= -128.0582$$

Iteration #1



$$T_0 = -100$$

$$T_1 = -128.0582$$

$$|\epsilon_a| = \left|\frac{T_1 - T_0}{T_1}\right|$$

$$= \left|\frac{-128.0582 \ - (-100)}{-128.0582}\right|$$

$$= 21.9105\ \%$$

Iteration #2

$$f(T) = -0.50598\,\text{x}10^{-10}T^3 - 0.38292\,\text{x}10^{-7}T^2 + 0.74363\,\text{x}10^{-4}T + 0.88318\,\text{x}10^{-2} = 0$$

$$f'(T) = -1.51794\,\text{x}\,10^{-10}T^2 + 0.76584\,\text{x}\,10^{-7}T + 0.74363\,\text{x}\,10^{-4}$$

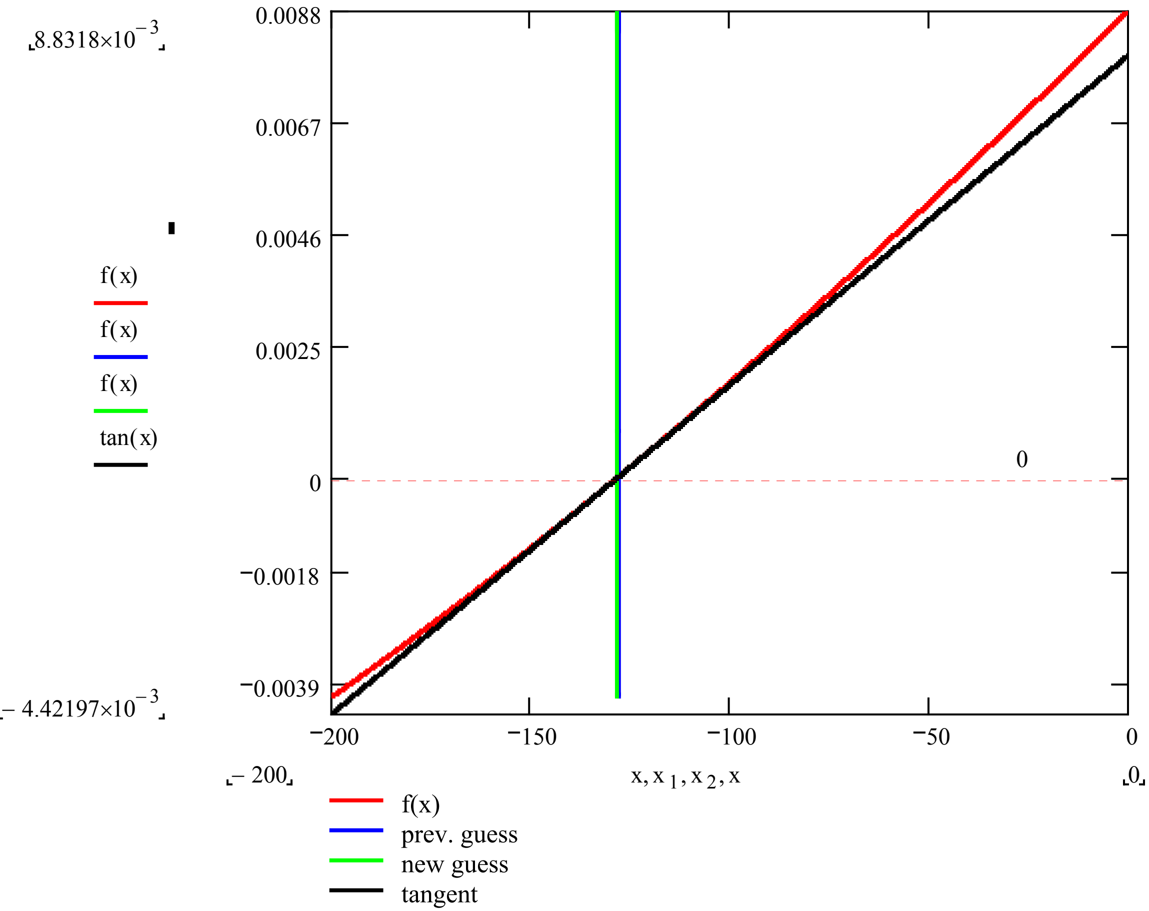
$$T_1 = -128.0582 \qquad T_2 = T_1 - \frac{f(T_1)}{f'(\text{T}_1)}$$

$$f(-128.0582) = -0.50598\text{x}10^{-10}(-128.0582)^3 - 0.38292\text{x}10^{-7}(-128.0582)^2$$
$$+ 0.74363\text{x}10^{-4}(-128.0582) + 0.88318\text{x}10^{-2} = 4.3214\text{x}10^{-5}$$

$$f'(-128.0582) = -1.51794\,\text{x}\,10^{-10}(-128.0582)^2 + 0.76584\,\text{x}\,10^{-7}(-128.0582)$$
$$+ 0.74363\,\text{x}\,10^{-4} = 6.2067\text{x}10^{-5}$$

$$T_2 = -128.0582 - \frac{4.3214\text{x}10^{-5}}{6.2067\,\text{x}10^{-5}}$$
$$= -128.7544$$

Iteration #2



$$T_2 = -128.7544$$

$$T_1 = -128.0582$$

$$\left|\in_a\right| = \left|\frac{T_2 - T_1}{T_2}\right|$$

$$= \left|\frac{-128.7544 - (-128.0582)}{-128.7544}\right|$$

$$= 0.5408\ \%$$

Iteration #3

$$f(T) = -0.50598 \text{ x}10^{-10} T^3 - 0.38292 \text{ x}10^{-7} T^2 + 0.74363 \text{ x}10^{-4} T + 0.88318 \text{ x}10^{-2} = 0$$

$$f'(T) = -1.51794 \text{ x } 10^{-10} T^2 + 0.76584 \text{ x } 10^{-7} T + 0.74363 \text{ x } 10^{-4}$$

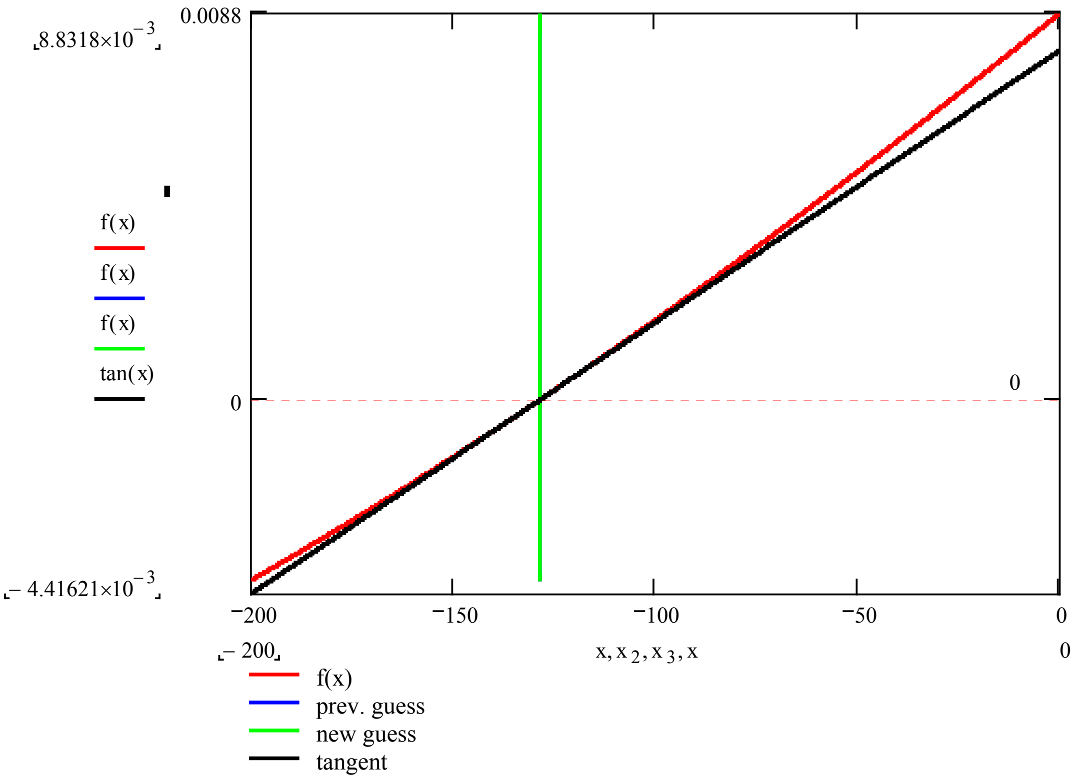
$$T_2 = -128.7544 \qquad T_3 = T_2 - \frac{f(T_2)}{f'(\mathrm{T}_2)}$$

$$f(-128.7544) = -0.50598\text{x}10^{-10}(-128.7544)^3 - 0.38292\text{x}10^{-7}(-128.7544)^2$$
$$+ 0.74363\text{x}10^{-4}(-128.7544) + 0.88318\text{x}10^{-2} = 2.8002\text{x}10^{-8}$$

$$f'(-128.7544) = -1.51794 \text{ x } 10^{-10}(-128.7544)^2 + 0.76584 \text{ x } 10^{-7}(-128.7544)$$
$$+ 0.74363 \text{ x } 10^{-4} = 6.9186\text{x}10^{-5}$$

$$T_3 = -128.7544 - \frac{2.8002\text{x}10^{-8}}{6.9186 \text{ x}10^{-5}}$$
$$= -128.7549$$

Iteration #3



$$T_3 = -128.7549$$

$$T_2 = -128.7544$$

$$\left|\in_a\right| = \left|\frac{T_3 - T_2}{T_3}\right|$$

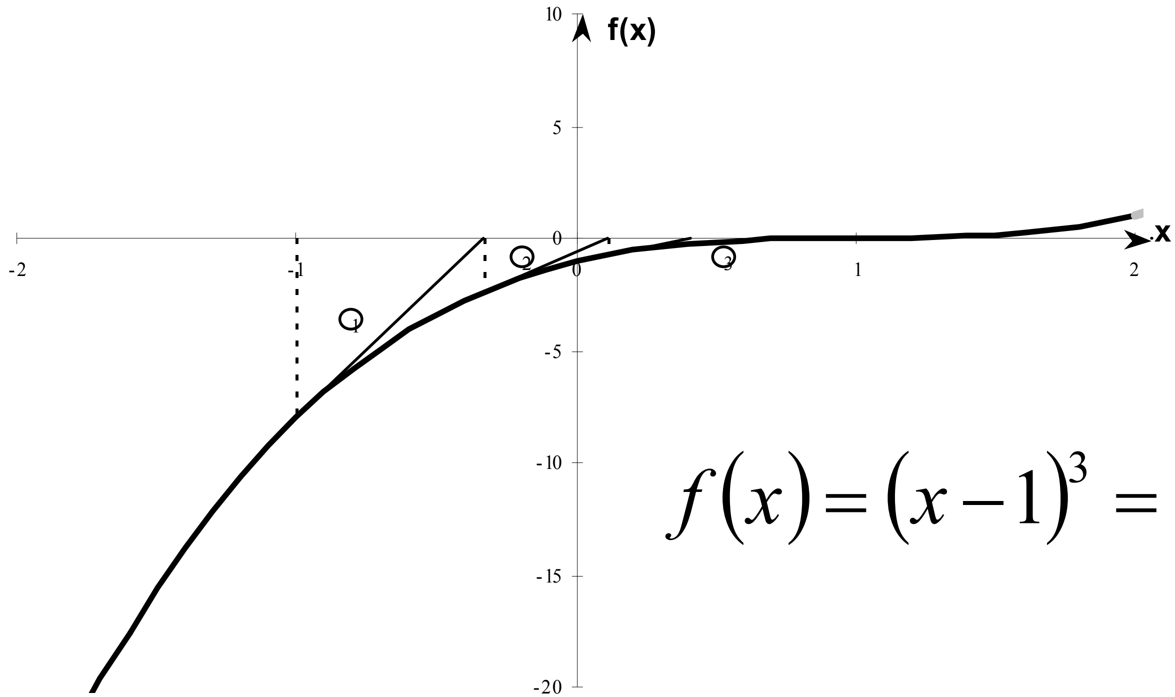
$$= \left|\frac{-128.7549 \ - (-128.7544 \ )}{-128.7549}\right|$$

$$= 3.5086 \text{ x}10^{-4}\%$$

Advantages

- Converges fast, if it converges
- Requires only one guess

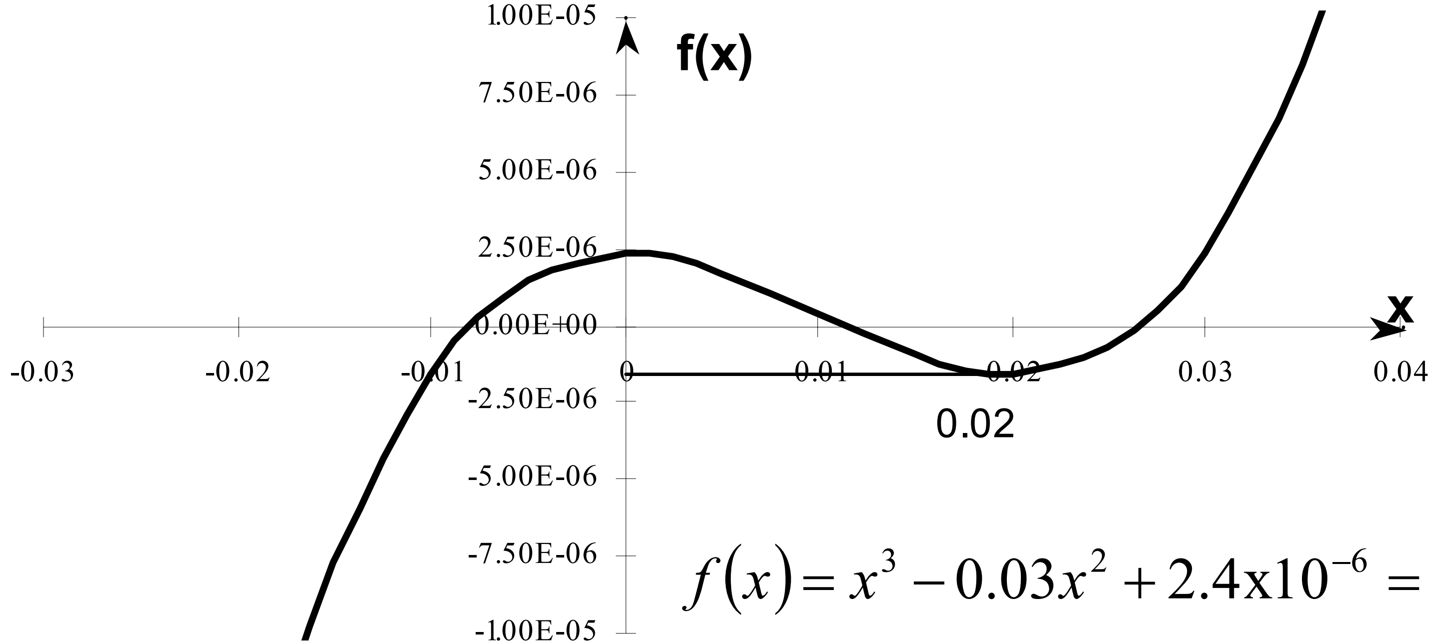
Drawbacks



$$f(x) = (x-1)^3 = 0$$

Inflection Point

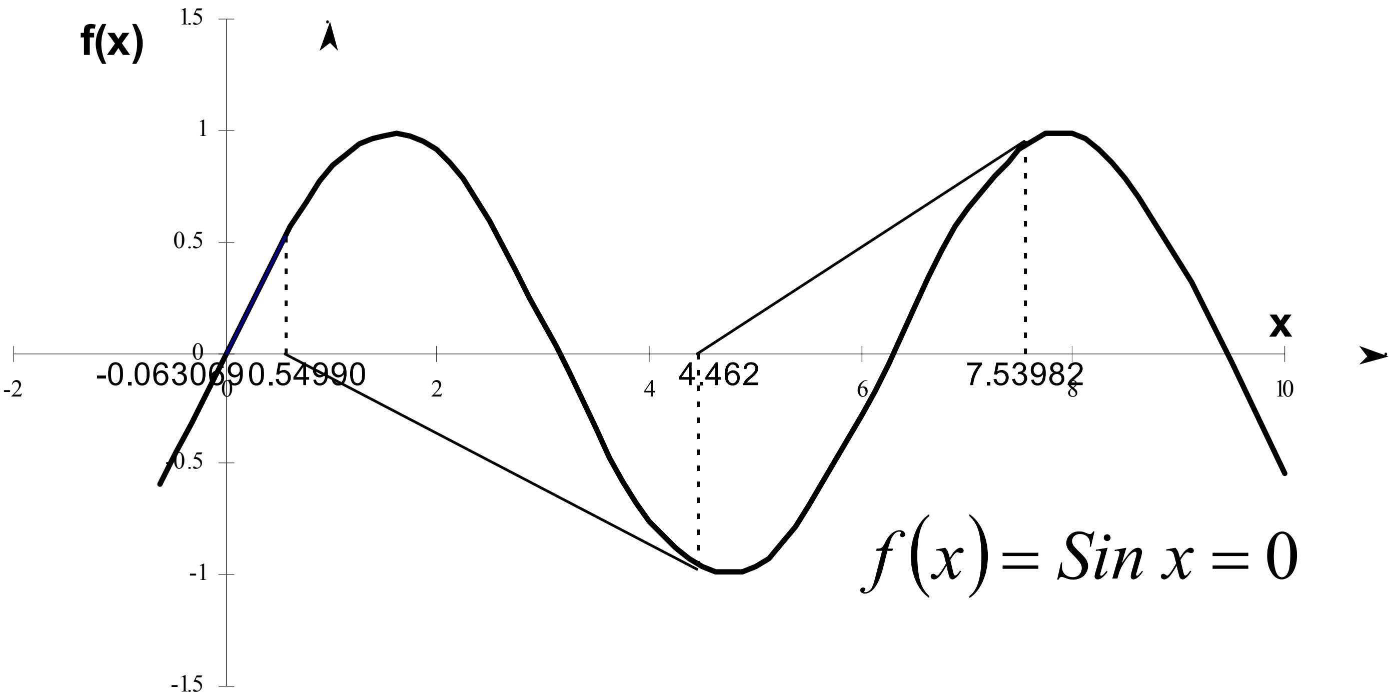
Drawbacks (continued)



$$f(x) = x^3 - 0.03x^2 + 2.4\text{x}10^{-6} = 0$$

Division by zero

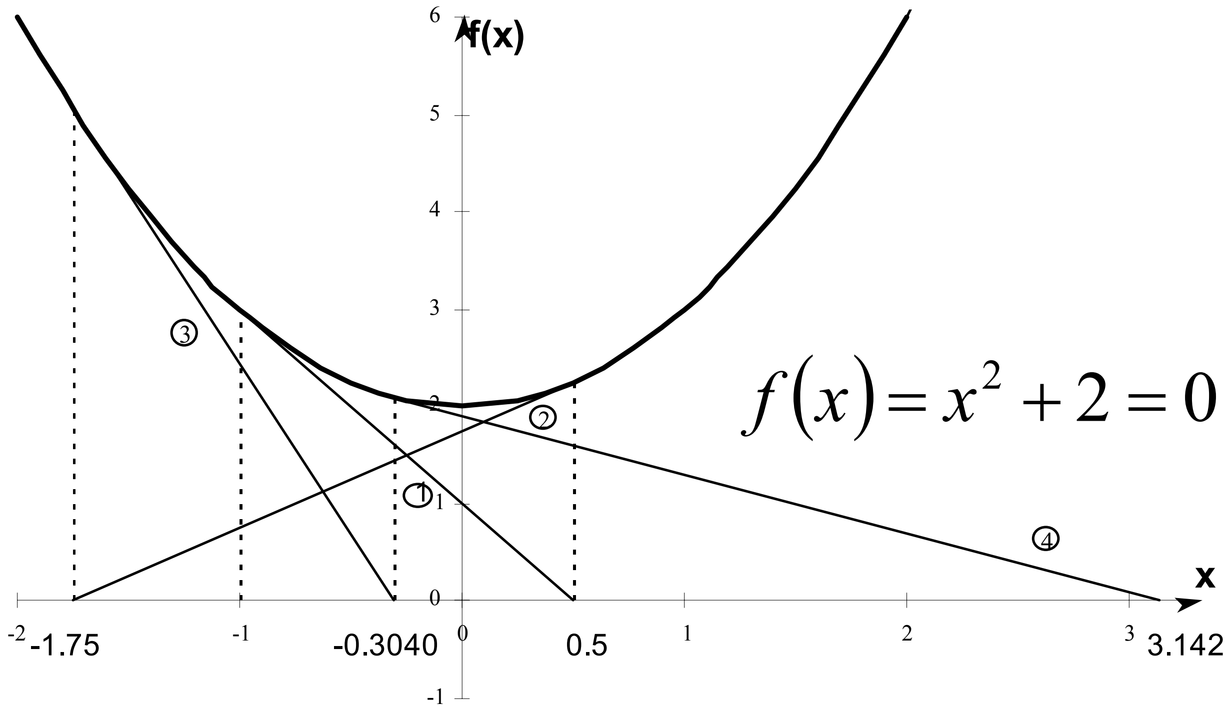
Drawbacks (continued)



$$f(x) = Sin\ x = 0$$

Root Jumping

Drawbacks (continued)



Oscillations near Local Maxima or Minima