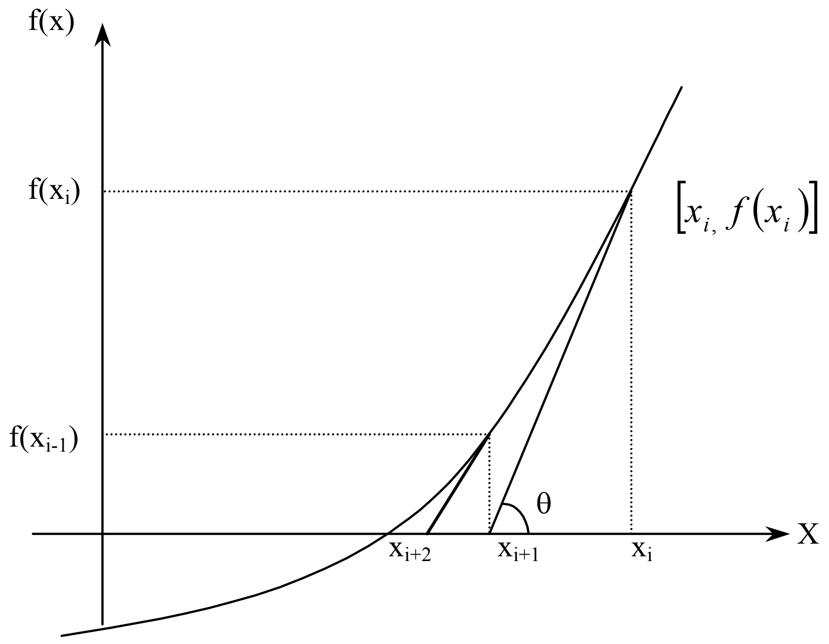
Roots of a Nonlinear Equation

Topic: Secant Method

Major: Electrical Engineering

Secant Method



Newton's Method

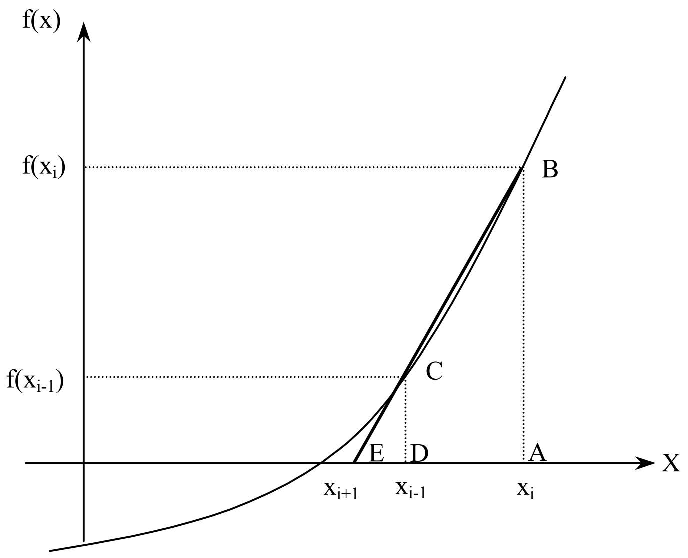
$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Approximate the derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Secant Method



Geometric Similar Triangles

$$\frac{AB}{AE} = \frac{DC}{DE}$$

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Algorithm for Secant Method

Step 1

Calculate the next estimate of the root from two initial guesses

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Find the absolute relative approximate error

$$\left|\in_a\right| = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \text{ x } 100$$

Step 2

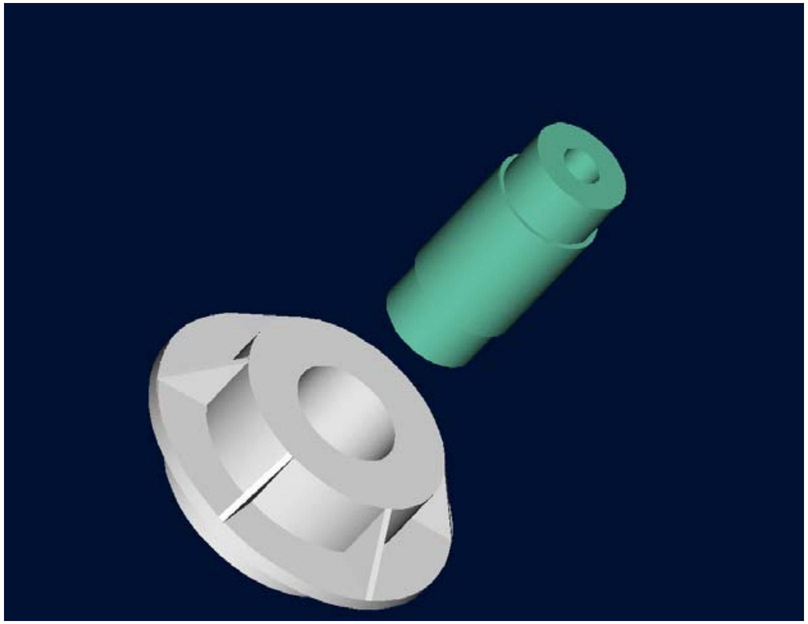
Find if the absolute relative approximate error is greater than the prespecified relative error tolerance.

If so, go back to step 1, else stop the algorithm.

Also check if the number of iterations has exceeded the maximum number of iterations.

Example

- A trunnion has to be cooled before it is shrink fit into a steel hub

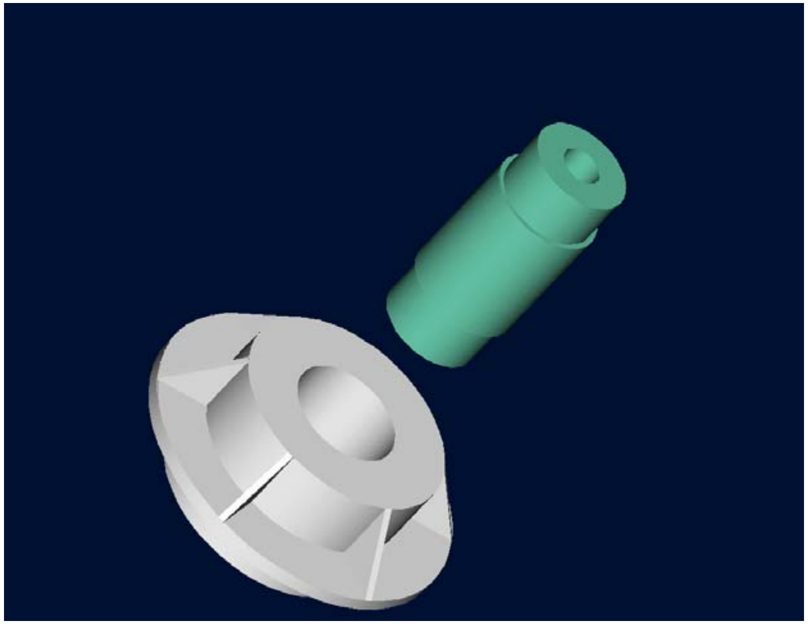


Solution

The equation that gives the temperature 'x' to which it has to be cooled to obtain the desired contraction is given by:

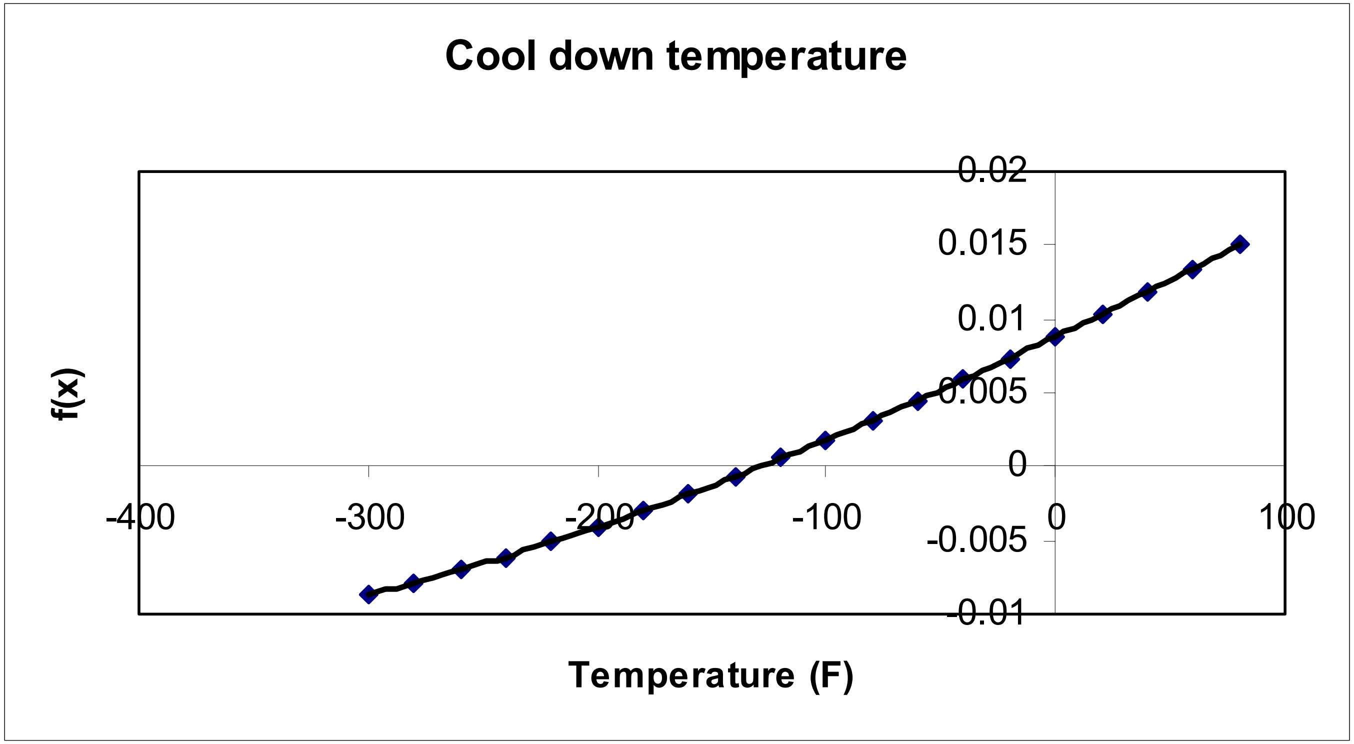
$$f(x) = -0.50598\text{x}10^{-10}x^3 + 0.38292\text{x}10^{-7}x^2 + 0.74363\text{x}10^{-4}x + 0.88318\text{x}10^{-2} = 0$$

Use the Secant method of finding roots of equations to find the temperature to which the trunnion should be cooled down to. Conduct three iterations.

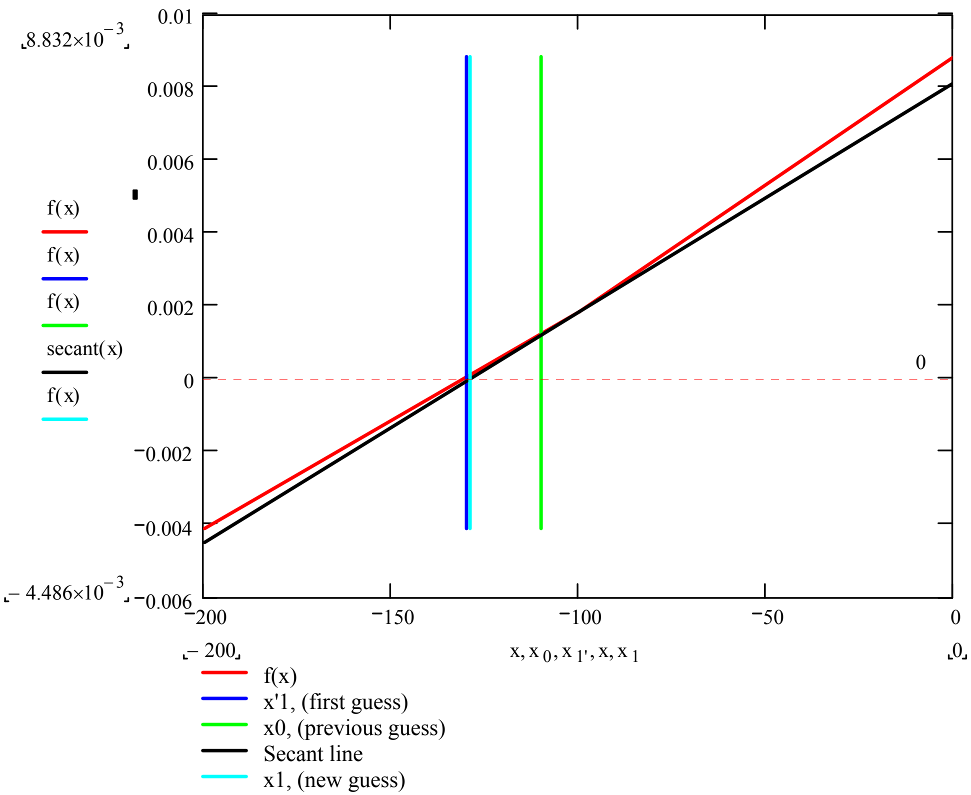


Graph of function f(x)

$$f(x) = -0.50598 \times 10^{-10} x^3 - 0.38292 \times 10^{-7} x^2 + 0.74363 \times 10^{-4} x + 0.88318 \times 10^{-2} = 0$$



Iteration #1



$$x_{-1} = -110, x_0 = -130$$

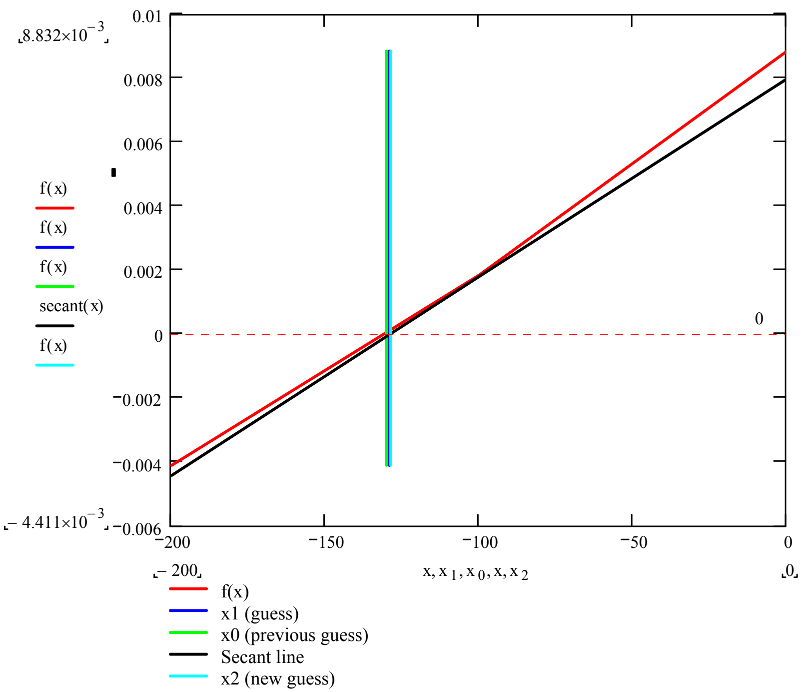
$$x_1 = x_0 - \frac{f(x_0)(x_0 - x_{-1})}{f(x_0) - f(x_{-1})}$$

$$x_1 = -130 - \frac{(0.01903)(-130 - (-110))}{(0.01903) - (1.1825\text{x}10^{-3})}$$

$$= -128.7660$$

$$|\in_a| = 0.95\%$$

Iteration #2



$$x_0 = -130, x_1 = -128.766$$

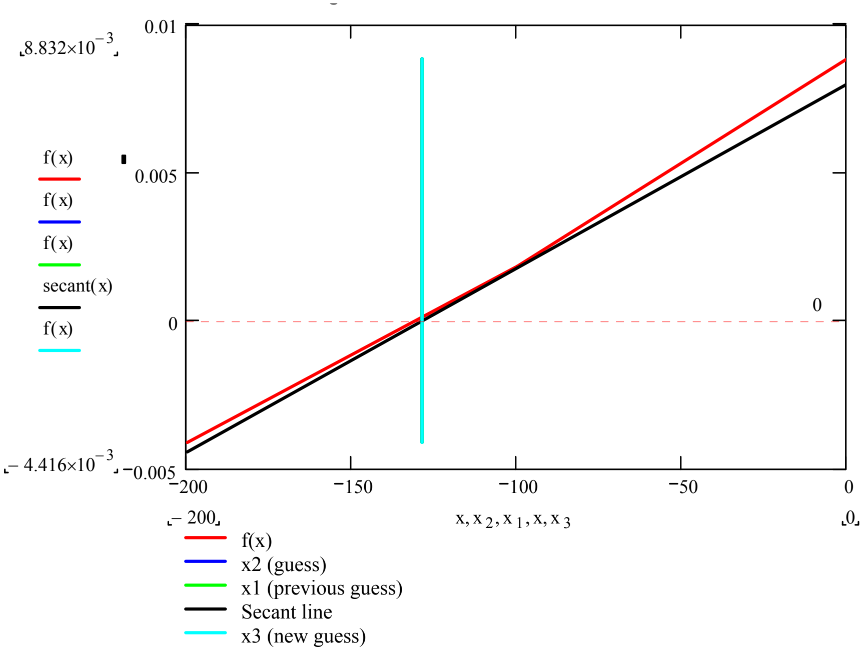
$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = -128.766 - \frac{1.3089\times10^{-6}(-128.766 - (-130))}{(1.3089\times10^{-6}) - (0.01903)}$$

$$= -128.7548$$

$$|\epsilon_a| = 0.01642\%$$

Iteration #3



$$x_1 = -128.766, x_2 = -128.7548$$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$x_3 = -128.7548 - \frac{(1.5241\text{x}10^{-9})(-128.7548 - (-128.766))}{(1.5241\text{x}10^{-9}) - (1.3089\text{x}10^{-6})}$$

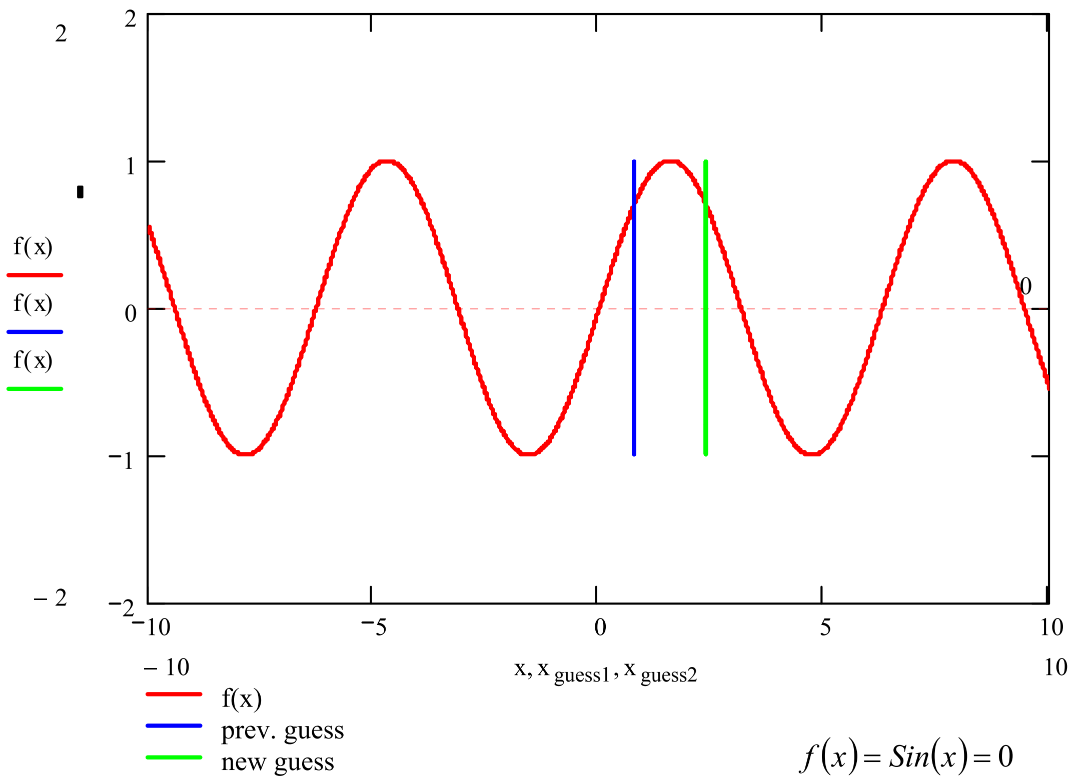
$$= -128.7549$$

$$|\in_a| = 1.9097\text{x}10^{-5}\%$$

Advantages

- Converges fast, if it converges
- Requires two guesses that do not need to bracket the root

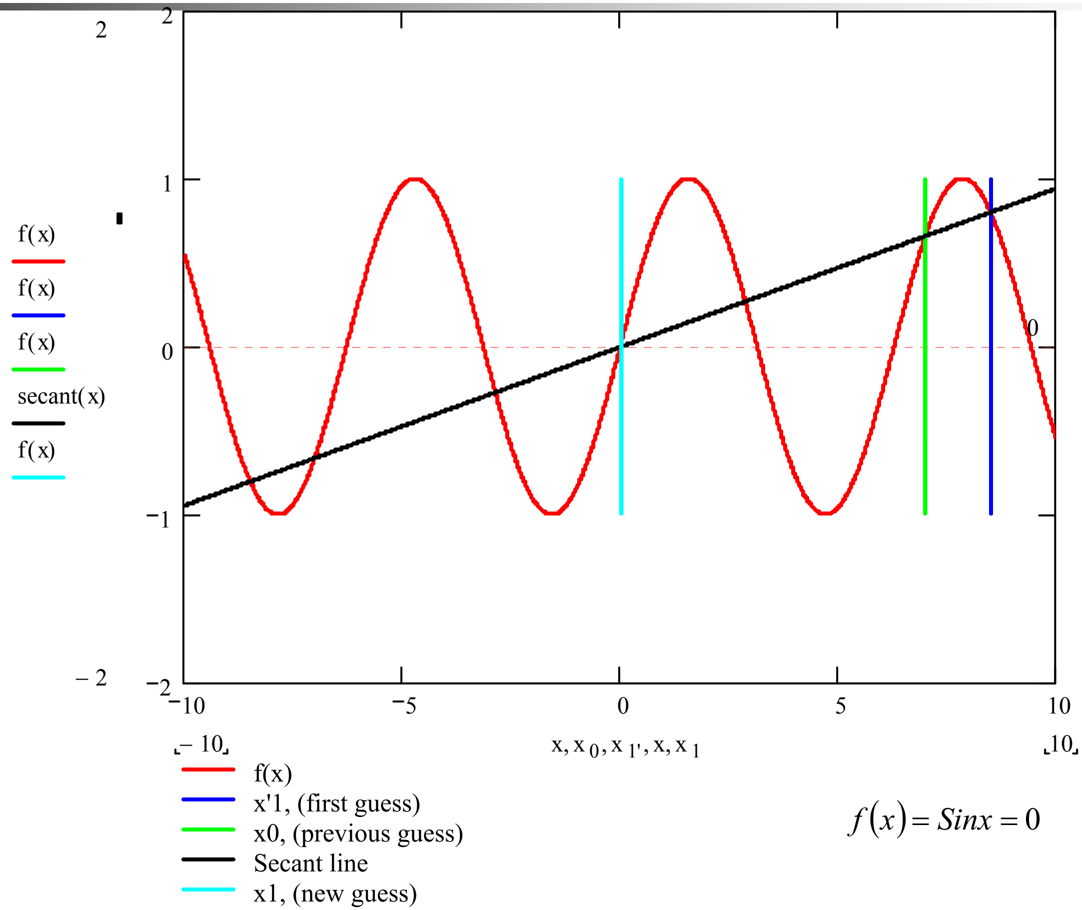
Drawbacks



$f(x) = Sin(x) = 0$

Division by zero

Drawbacks (continued)



$$f(x) = Sinx = 0$$

Root Jumping