

## Chapter 05.03

# Newton's Divided Difference Interpolation

*After reading this chapter, you should be able to:*

1. *derive Newton's divided difference method of interpolation,*
2. *apply Newton's divided difference method of interpolation, and*
3. *apply Newton's divided difference method interpolants to find derivatives and integrals.*

### What is interpolation?

Many a times, data is given only at discrete points such as  $(x_0, y_0)$ ,  $(x_1, y_1)$ ,  $\dots$ ,  $(x_{n-1}, y_{n-1})$ ,  $(x_n, y_n)$ . So, how then does one find the value of  $y$  at any other value of  $x$ ? Well, a continuous function  $f(x)$  may be used to represent the  $n+1$  data values with  $f(x)$  passing through the  $n+1$  points (Figure 1). Then one can find the value of  $y$  at any other value of  $x$ . This is called interpolation.

Of course, if  $x$  falls outside the range of  $x$  for which the data is given, it is no longer interpolation but instead is called extrapolation.

So what kind of function  $f(x)$  should one choose? A polynomial is a common choice for an interpolating function because polynomials are easy to

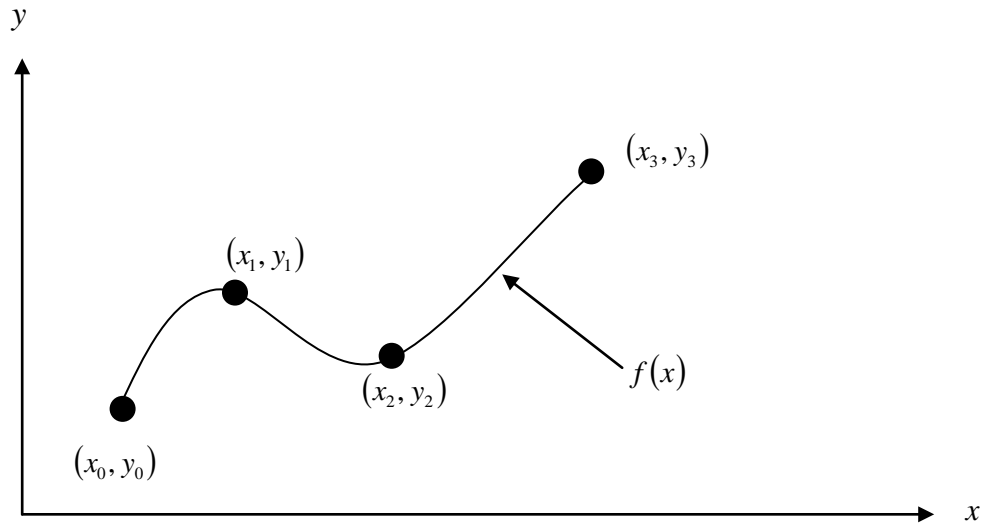
- (A) evaluate,
- (B) differentiate, and
- (C) integrate,

relative to other choices such as a trigonometric and exponential series.

Polynomial interpolation involves finding a polynomial of order  $n$  that passes through the  $n+1$  points. One of the methods of interpolation is called Newton's divided difference polynomial method. Other methods include the direct method and the Lagrangian interpolation method. We discuss Newton's divided difference polynomial method in this chapter.

### Newton's Divided Difference Polynomial Method

To illustrate this method, linear and quadratic interpolation is presented first. Then, the general form of Newton's divided difference polynomial method is presented. To illustrate the general form, cubic interpolation is shown in Figure 1.



**Figure 1** Interpolation of discrete data.

### Linear Interpolation

Given  $(x_0, y_0)$  and  $(x_1, y_1)$ , fit a linear interpolant through the data. Noting  $y = f(x)$  and  $y_1 = f(x_1)$ , assume the linear interpolant  $f_1(x)$  is given by (Figure 2)

$$f_1(x) = b_0 + b_1(x - x_0)$$

Since at  $x = x_0$ ,

$$f_1(x_0) = f(x_0) = b_0 + b_1(x_0 - x_0) = b_0$$

and at  $x = x_1$ ,

$$\begin{aligned} f_1(x_1) &= f(x_1) = b_0 + b_1(x_1 - x_0) \\ &= f(x_0) + b_1(x_1 - x_0) \end{aligned}$$

giving

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

So

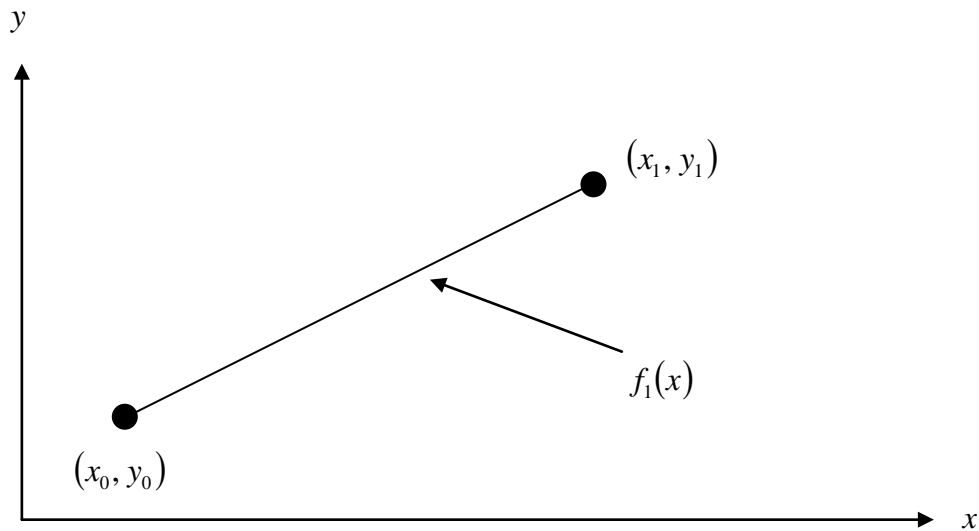
$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

giving the linear interpolant as

$$f_1(x) = b_0 + b_1(x - x_0)$$

$$f_1(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$



**Figure 2** Linear interpolation.

**Example 1**

For the purpose of shrinking a trunnion into a hub, the reduction of diameter  $\Delta D$  of a trunnion shaft by cooling it through a temperature change of  $\Delta T$  is given by

$$\Delta D = D\alpha\Delta T$$

where

$D$  = original diameter (in.)

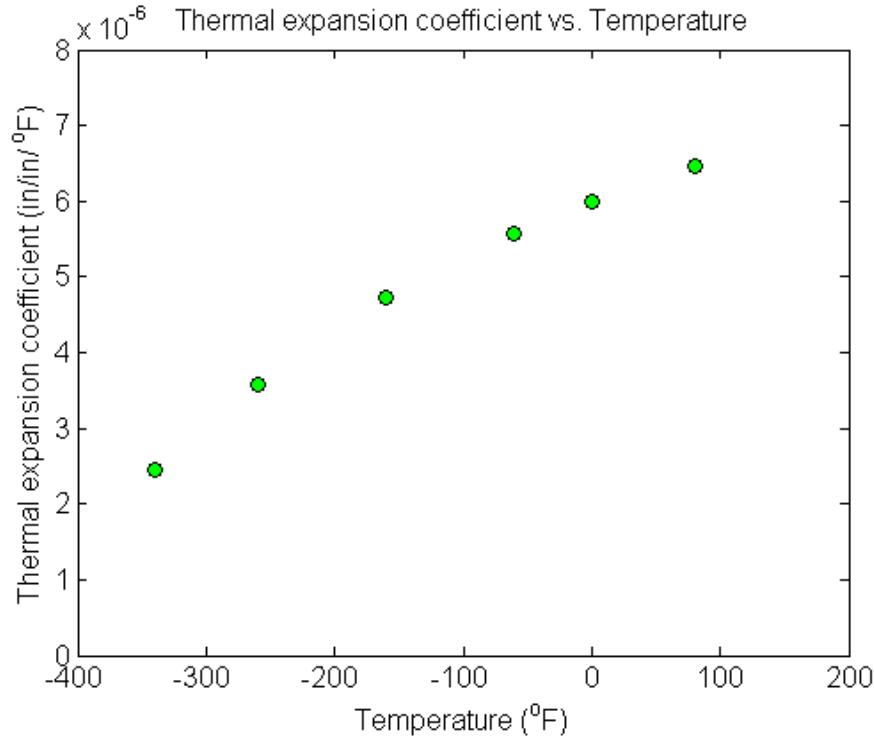
$\alpha$  = coefficient of thermal expansion at average temperature (in/in/°F)

The trunnion is cooled from 80°F to -108°F, giving the average temperature as -14°F.

The table of the coefficient of thermal expansion vs. temperature data is given in Table 1.

**Table 1** Thermal expansion coefficient as a function of temperature.

Temperature, $T$ (°F)	Thermal Expansion Coefficient, $\alpha$ (in/in/°F)
80	$6.47 \times 10^{-6}$
0	$6.00 \times 10^{-6}$
-60	$5.58 \times 10^{-6}$
-160	$4.72 \times 10^{-6}$
-260	$3.58 \times 10^{-6}$
-340	$2.45 \times 10^{-6}$



**Figure 3** Thermal expansion coefficient vs. temperature.

Determine the value of the coefficient of thermal expansion at  $T = -14^\circ\text{F}$  using Newton's divided difference method of interpolation and a first order polynomial.

### Solution

For linear interpolation, the coefficient of thermal expansion is given by

$$\alpha(T) = b_0 + b_1(T - T_0)$$

Since we want to find the coefficient of thermal expansion at  $T = -14$  and we are using a first order polynomial, we need to choose the two data points that are closest to  $T = -14$  that also bracket  $T = -14$  to evaluate it. The two points are  $T_0 = 0$  and  $T_1 = -60$ .

Then

$$T_0 = 0, \quad \alpha(T_0) = 6.00 \times 10^{-6}$$

$$T_1 = -60, \quad \alpha(T_1) = 5.58 \times 10^{-6}$$

gives

$$b_0 = \alpha(T_0)$$

$$= 6.00 \times 10^{-6}$$

$$b_1 = \frac{\alpha(T_1) - \alpha(T_0)}{T_1 - T_0}$$

$$= \frac{5.58 \times 10^{-6} - 6.00 \times 10^{-6}}{-60 - 0}$$

$$= 0.007 \times 10^{-6}$$

Hence

$$\begin{aligned}\alpha(T) &= b_0 + b_1(T - T_0) \\ &= 6.00 \times 10^{-6} + 0.007 \times 10^{-6}(T - 0), \quad -60 \leq T \leq 0\end{aligned}$$

At  $T = -14$

$$\begin{aligned}\alpha(-14) &= 6.00 \times 10^{-6} + 0.007 \times 10^{-6}(-14 - 0) \\ &= 5.902 \times 10^{-6} \text{ in/in/}^\circ\text{F}\end{aligned}$$

If we expand

$$\alpha(T) = 6.00 \times 10^{-6} + 0.007 \times 10^{-6}(T - 0), \quad -60 \leq T \leq 0$$

we get

$$\alpha(T) = 6.00 \times 10^{-6} + 0.007 \times 10^{-6}T, \quad -60 \leq T \leq 0$$

This is the same expression that was obtained with the direct method.

### Quadratic Interpolation

Given  $(x_0, y_0)$ ,  $(x_1, y_1)$ , and  $(x_2, y_2)$ , fit a quadratic interpolant through the data. Noting  $y = f(x)$ ,  $y_0 = f(x_0)$ ,  $y_1 = f(x_1)$ , and  $y_2 = f(x_2)$ , assume the quadratic interpolant  $f_2(x)$  is given by

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

At  $x = x_0$ ,

$$\begin{aligned}f_2(x_0) &= f(x_0) = b_0 + b_1(x_0 - x_0) + b_2(x_0 - x_0)(x_0 - x_1) \\ &= b_0\end{aligned}$$

$$b_0 = f(x_0)$$

At  $x = x_1$

$$\begin{aligned}f_2(x_1) &= f(x_1) = b_0 + b_1(x_1 - x_0) + b_2(x_1 - x_0)(x_1 - x_1) \\ f(x_1) &= f(x_0) + b_1(x_1 - x_0)\end{aligned}$$

giving

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

At  $x = x_2$

$$\begin{aligned}f_2(x_2) &= f(x_2) = b_0 + b_1(x_2 - x_0) + b_2(x_2 - x_0)(x_2 - x_1) \\ f(x_2) &= f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x_2 - x_0) + b_2(x_2 - x_0)(x_2 - x_1)\end{aligned}$$

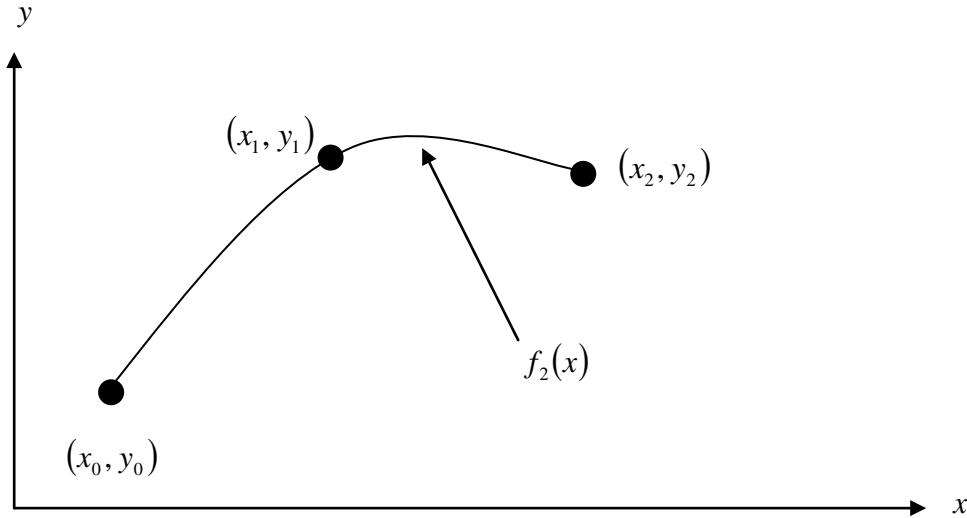
Giving

$$b_2 = \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

Hence the quadratic interpolant is given by

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$= f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0) + \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}(x - x_0)(x - x_1)$$



**Figure 4** Quadratic interpolation.

### Example 2

For the purpose of shrinking a trunnion into a hub, the reduction of diameter  $\Delta D$  of a trunnion shaft by cooling it through a temperature change of  $\Delta T$  is given by

$$\Delta D = D\alpha\Delta T$$

where

$D$  = original diameter (in.)

$\alpha$  = coefficient of thermal expansion at average temperature (in/in/°F)

The trunnion is cooled from 80°F to -108°F, giving the average temperature as -14°F.

The table of the coefficient of thermal expansion vs. temperature data is given in Table 2.

**Table 2** Thermal expansion coefficient as a function of temperature.

Temperature, $T$ (°F)	Thermal Expansion Coefficient, $\alpha$ (in/in/°F)
80	$6.47 \times 10^{-6}$
0	$6.00 \times 10^{-6}$
-60	$5.58 \times 10^{-6}$
-160	$4.72 \times 10^{-6}$
-260	$3.58 \times 10^{-6}$
-340	$2.45 \times 10^{-6}$

Determine the value of the coefficient of thermal expansion at  $T = -14^\circ\text{F}$  using Newton's divided difference method of interpolation and a second order polynomial. Find the absolute relative approximate error for the second order polynomial approximation.

**Solution**

For quadratic interpolation, the coefficient of thermal expansion is given by

$$\alpha(T) = b_0 + b_1(T - T_0) + b_2(T - T_0)(T - T_1)$$

Since we want to find the coefficient of thermal expansion at  $T = -14$ , we need to choose the three data points that are closest to  $T = -14$  that also bracket  $T = -14$  to evaluate it. The three points are  $T_0 = 80$ ,  $T_1 = 0$  and  $T_2 = -60$ .

Then

$$T_0 = 80, \quad \alpha(T_0) = 6.47 \times 10^{-6}$$

$$T_1 = 0, \quad \alpha(T_1) = 6.00 \times 10^{-6}$$

$$T_2 = -60, \quad \alpha(T_2) = 5.58 \times 10^{-6}$$

gives

$$b_0 = \alpha(T_0)$$

$$= 6.47 \times 10^{-6}$$

$$b_1 = \frac{\alpha(T_1) - \alpha(T_0)}{T_1 - T_0}$$

$$= \frac{6.00 \times 10^{-6} - 6.47 \times 10^{-6}}{0 - 80}$$

$$= 5.875 \times 10^{-9}$$

$$b_2 = \frac{\frac{\alpha(T_2) - \alpha(T_1)}{T_2 - T_1} - \frac{\alpha(T_1) - \alpha(T_0)}{T_1 - T_0}}{T_2 - T_0}$$

$$= \frac{\frac{5.58 \times 10^{-6} - 6.00 \times 10^{-6}}{-60 - 0} - \frac{6.00 \times 10^{-6} - 6.47 \times 10^{-6}}{0 - 80}}{-60 - 80}$$

$$= \frac{0.007 \times 10^{-6} - 0.005875 \times 10^{-6}}{-140}$$

$$= -8.0357 \times 10^{-12}$$

Hence

$$\alpha(T) = b_0 + b_1(T - T_0) + b_2(T - T_0)(T - T_1)$$

$$= 6.47 \times 10^{-6} + 5.875 \times 10^{-9}(T - 80) - 8.0357 \times 10^{-12}(T - 80)(T - 0), \quad -60 \leq T \leq 80$$

At  $T = -14$ ,

$$\alpha(-14) = 6.47 \times 10^{-6} + 5.875 \times 10^{-9}(-14 - 80) - 8.0357 \times 10^{-12}(-14 - 80)(-14 - 0)$$

$$= 5.9072 \times 10^{-6} \text{ in/in/}^\circ\text{F}$$

The absolute relative approximate error  $|\epsilon_a|$  obtained between the results from the first and second order polynomial is

$$|\epsilon_a| = \left| \frac{5.9072 \times 10^{-6} - 5.902 \times 10^{-6}}{5.9072 \times 10^{-6}} \right| \times 100$$

$$= 0.087605\%$$

If we expand

$$\alpha(T) = 6.47 \times 10^{-6} + 5.875 \times 10^{-9}(T - 80) - 8.0357 \times 10^{-12}(T - 80)(T - 0), \quad -60 \leq T \leq 80$$

we get

$$\alpha(T) = 6.00 \times 10^{-6} + 6.5179 \times 10^{-9}T - 8.0357 \times 10^{-12}T^2, \quad -60 \leq T \leq 80$$

This is the same expression that was obtained with the direct method.

### General Form of Newton's Divided Difference Polynomial

In the two previous cases, we found linear and quadratic interpolants for Newton's divided difference method. Let us revisit the quadratic polynomial interpolant formula

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

where

$$\begin{aligned} b_0 &= f(x_0) \\ b_1 &= \frac{f(x_1) - f(x_0)}{x_1 - x_0} \\ b_2 &= \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0} \end{aligned}$$

Note that  $b_0$ ,  $b_1$ , and  $b_2$  are finite divided differences.  $b_0$ ,  $b_1$ , and  $b_2$  are the first, second, and third finite divided differences, respectively. We denote the first divided difference by

$$f[x_0] = f(x_0)$$

the second divided difference by

$$f[x_1, x_0] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

and the third divided difference by

$$\begin{aligned} f[x_2, x_1, x_0] &= \frac{f[x_2, x_1] - f[x_1, x_0]}{x_2 - x_0} \\ &= \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0} \end{aligned}$$

where  $f[x_0]$ ,  $f[x_1, x_0]$ , and  $f[x_2, x_1, x_0]$  are called bracketed functions of their variables enclosed in square brackets.

Rewriting,

$$f_2(x) = f[x_0] + f[x_1, x_0](x - x_0) + f[x_2, x_1, x_0](x - x_0)(x - x_1)$$

This leads us to writing the general form of the Newton's divided difference polynomial for  $n + 1$  data points,  $(x_0, y_0), (x_1, y_1), \dots, (x_{n-1}, y_{n-1}), (x_n, y_n)$ , as

$$f_n(x) = b_0 + b_1(x - x_0) + \dots + b_n(x - x_0)(x - x_1)\dots(x - x_{n-1})$$

where

$$\begin{aligned}
 b_0 &= f[x_0] \\
 b_1 &= f[x_1, x_0] \\
 b_2 &= f[x_2, x_1, x_0] \\
 &\vdots \\
 b_{n-1} &= f[x_{n-1}, x_{n-2}, \dots, x_0] \\
 b_n &= f[x_n, x_{n-1}, \dots, x_0]
 \end{aligned}$$

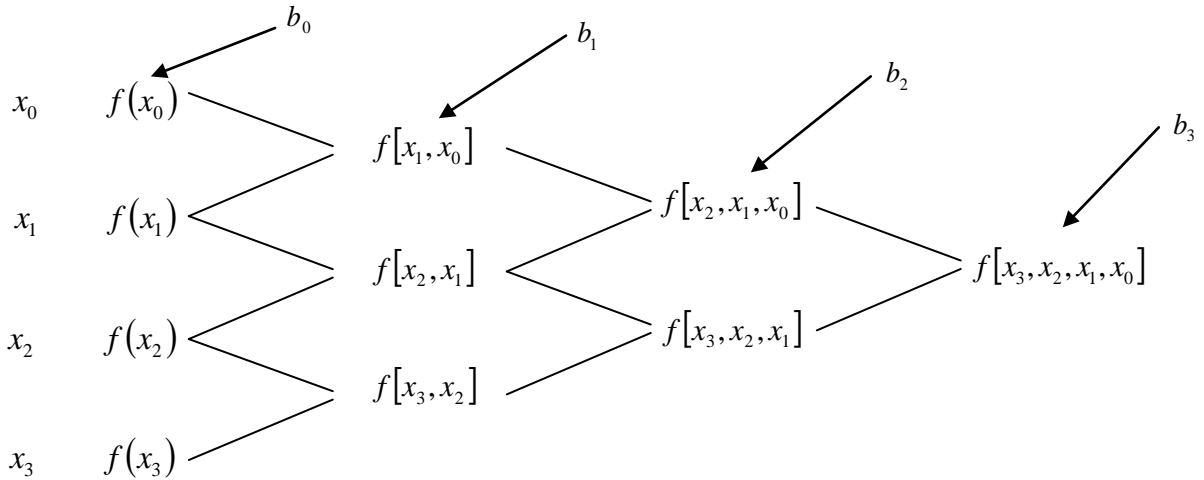
where the definition of the  $m^{\text{th}}$  divided difference is

$$\begin{aligned}
 b_m &= f[x_m, \dots, x_0] \\
 &= \frac{f[x_m, \dots, x_1] - f[x_{m-1}, \dots, x_0]}{x_m - x_0}
 \end{aligned}$$

From the above definition, it can be seen that the divided differences are calculated recursively.

For an example of a third order polynomial, given  $(x_0, y_0)$ ,  $(x_1, y_1)$ ,  $(x_2, y_2)$ , and  $(x_3, y_3)$ ,

$$\begin{aligned}
 f_3(x) &= f[x_0] + f[x_1, x_0](x - x_0) + f[x_2, x_1, x_0](x - x_0)(x - x_1) \\
 &\quad + f[x_3, x_2, x_1, x_0](x - x_0)(x - x_1)(x - x_2)
 \end{aligned}$$



**Figure 5** Table of divided differences for a cubic polynomial.

**Example 3**

For the purpose of shrinking a trunnion into a hub, the reduction of diameter  $\Delta D$  of a trunnion shaft by cooling it through a temperature change of  $\Delta T$  is given by

$$\Delta D = D\alpha\Delta T$$

where

$D$  = original diameter (in.)

$\alpha$  = coefficient of thermal expansion at average temperature (in/in/°F)

The trunnion is cooled from 80°F to -108°F, giving the average temperature as -14°F.

The table of the coefficient of thermal expansion vs. temperature data is given in Table 3.

**Table 3** Thermal expansion coefficient as a function of temperature.

Temperature, $T$ ( $^{\circ}\text{F}$ )	Thermal Expansion Coefficient, $\alpha$ (in/in/ $^{\circ}\text{F}$ )
80	$6.47 \times 10^{-6}$
0	$6.00 \times 10^{-6}$
-60	$5.58 \times 10^{-6}$
-160	$4.72 \times 10^{-6}$
-260	$3.58 \times 10^{-6}$
-340	$2.45 \times 10^{-6}$

- a) Determine the value of the coefficient of thermal expansion at  $T = -14^{\circ}\text{F}$  using Newton's divided difference method of interpolation and a third order polynomial. Find the absolute relative approximate error for the third order polynomial approximation.
- b) The actual reduction in diameter is given by

$$\Delta D = D \int_{T_r}^{T_f} \alpha dT$$

where

$T_r$  = room temperature ( $^{\circ}\text{F}$ )

$T_f$  = temperature of cooling medium ( $^{\circ}\text{F}$ )

Since

$$T_r = 80^{\circ}\text{F}$$

$$T_f = -108^{\circ}\text{F}$$

$$\Delta D = D \int_{80}^{-108} \alpha dT$$

Find out the percentage difference in the reduction in the diameter by the above integral formula and the result using the thermal expansion coefficient from part (a).

### Solution

- a) For cubic interpolation, the coefficient of thermal expansion is given by

$$\alpha(T) = b_0 + b_1(T - T_0) + b_2(T - T_0)(T - T_1) + b_3(T - T_0)(T - T_1)(T - T_2)$$

Since we want to find the coefficient of thermal expansion at  $T = -14$  and we are using a third order polynomial, we need to choose the four data points that are closest to  $T = -14$  and bracket  $T = -14$ . These four data points are  $T_0 = 80$ ,  $T_1 = 0$ ,  $T_2 = -60$  and  $T_3 = -160$ .

Then

$$T_0 = 80, \quad \alpha(T_0) = 6.47 \times 10^{-6}$$

$$T_1 = 0, \quad \alpha(T_1) = 6.00 \times 10^{-6}$$

$$T_2 = -60, \quad \alpha(T_2) = 5.58 \times 10^{-6}$$

$$T_3 = -160, \quad \alpha(T_3) = 4.72 \times 10^{-6}$$

gives

$$\begin{aligned}
 b_0 &= \alpha[T_0] \\
 &= \alpha(T_0) \\
 &= 6.47 \times 10^{-6} \\
 b_1 &= \alpha[T_1, T_0] \\
 &= \frac{\alpha(T_1) - \alpha(T_0)}{T_1 - T_0} \\
 &= \frac{6.00 \times 10^{-6} - 6.47 \times 10^{-6}}{0 - 80} \\
 &= 5.875 \times 10^{-9} \\
 b_2 &= \alpha[T_2, T_1, T_0] \\
 &= \frac{\alpha[T_2, T_1] - \alpha[T_1, T_0]}{T_2 - T_0} \\
 \alpha[T_2, T_1] &= \frac{\alpha(T_2) - \alpha(T_1)}{T_2 - T_1} \\
 &= \frac{5.58 \times 10^{-6} - 6.00 \times 10^{-6}}{-60 - 0} \\
 &= 0.007 \times 10^{-6} \\
 \alpha[T_1, T_0] &= 5.875 \times 10^{-9} \\
 b_2 &= \frac{\alpha[T_2, T_1] - \alpha[T_1, T_0]}{T_2 - T_0} \\
 &= \frac{0.007 \times 10^{-6} - 0.005875 \times 10^{-6}}{-60 - 80} \\
 &= -8.0357 \times 10^{-12} \\
 b_3 &= \alpha[T_3, T_2, T_1, T_0] \\
 &= \frac{\alpha[T_3, T_2, T_1] - \alpha[T_2, T_1, T_0]}{T_3 - T_0} \\
 \alpha[T_3, T_2, T_1] &= \frac{\alpha[T_3, T_2] - \alpha[T_2, T_1]}{T_3 - T_1} \\
 \alpha[T_3, T_2] &= \frac{\alpha(T_3) - \alpha(T_2)}{T_3 - T_2} \\
 &= \frac{4.72 \times 10^{-6} - 5.58 \times 10^{-6}}{-160 + 60} \\
 &= 0.0086 \times 10^{-6} \\
 \alpha[T_2, T_1] &= 0.007 \times 10^{-6}
 \end{aligned}$$

$$\begin{aligned}
\alpha[T_3, T_2, T_1] &= \frac{\alpha[T_3, T_2] - \alpha[T_2, T_1]}{T_3 - T_1} \\
&= \frac{0.0086 \times 10^{-6} - 0.007 \times 10^{-6}}{-160 - 0} \\
&= -10^{-11} \\
\alpha[T_2, T_1, T_0] &= -8.0357 \times 10^{-12} \\
b_3 &= \alpha[T_3, T_2, T_1, T_0] \\
&= \frac{\alpha[T_3, T_2, T_1] - \alpha[T_2, T_1, T_0]}{T_3 - T_0} \\
&= \frac{-10^{-11} + 8.0357 \times 10^{-12}}{-160 - 80} \\
&= 8.1845 \times 10^{-15}
\end{aligned}$$

Hence

$$\begin{aligned}
\alpha(T) &= b_0 + b_1(T - T_0) + b_2(T - T_0)(T - T_1) + b_3(T - T_0)(T - T_1)(T - T_2) \\
&= 6.47 \times 10^{-6} + 5.875 \times 10^{-9}(T - 80) - 8.0357 \times 10^{-12}(T - 80)(T - 0) \\
&\quad + 8.1845 \times 10^{-15}(T - 80)(T - 0)(T + 60)
\end{aligned}$$

At  $T = -14$ ,

$$\begin{aligned}
\alpha(-14) &= 6.47 \times 10^{-6} + 5.875 \times 10^{-9}(-14 - 80) - 8.0357 \times 10^{-12}(-14 - 80)(-14 - 0) \\
&\quad + 8.1845 \times 10^{-15}(-14 - 80)(-14 - 0)(-14 + 60) \\
&= 5.9077 \times 10^{-6} \text{ in/in/}^\circ\text{F}
\end{aligned}$$

The absolute relative approximate error  $|\epsilon_a|$  obtained between the results from the second and third order polynomial is

$$\begin{aligned}
|\epsilon_a| &= \left| \frac{5.9077 \times 10^{-6} - 5.9072 \times 10^{-6}}{5.9077 \times 10^{-6}} \right| \times 100 \\
&= 0.0083867\%
\end{aligned}$$

b) In finding the percentage difference in the reduction in diameter, we can rearrange the integral formula to

$$\frac{\Delta D}{D} = \int_{T_r}^{T_f} \alpha dT$$

and since we know from part (a) that

$$\begin{aligned}
\alpha(T) &= 6.47 \times 10^{-6} + 5.875 \times 10^{-9}(T - 80) - 8.0357 \times 10^{-12}(T - 80)(T - 0) \\
&\quad + 8.1845 \times 10^{-15}(T - 80)(T - 0)(T + 60), \quad -160 \leq T \leq 80
\end{aligned}$$

Combining like terms, we get

$$\alpha(T) = 6.00 \times 10^{-6} + 6.4786 \times 10^{-9}T - 8.1994 \times 10^{-12}T^2 + 8.1845 \times 10^{-15}T^3, \quad -160 \leq T \leq 80$$

We see that we can use the integral formula in the range from  $T_f = -108^\circ\text{F}$  to  $T_r = 80^\circ\text{F}$

Therefore,

$$\begin{aligned}
\frac{\Delta D}{D} &= \int_{T_r}^{T_f} \alpha dT \\
&= \int_{80}^{-108} (6.00 \times 10^{-6} + 6.4786 \times 10^{-3} T - 8.1994 \times 10^{-6} T^2 + 8.1845 \times 10^{-9} T^3) dT \\
&= \left[ 6.00 \times 10^{-6} T + 6.4786 \times 10^{-9} \frac{T^2}{2} - 8.1994 \times 10^{-9} \frac{T^3}{3} + 8.1845 \times 10^{-15} \frac{T^4}{4} \right]_{80}^{-108} \\
&= -1105.9 \times 10^{-6}
\end{aligned}$$

So  $\frac{\Delta D}{D} = -1105.9 \times 10^{-6}$  in/in using the actual reduction in diameter integral formula. If we use the average value for the coefficient of thermal expansion from part (a), we get

$$\begin{aligned}
\frac{\Delta D}{D} &= \alpha \Delta T \\
&= \alpha (T_f - T_r) \\
&= 5.9077 \times 10^{-6} (-108 - 80) \\
&= -1110.6 \times 10^{-6}
\end{aligned}$$

and  $\frac{\Delta D}{D} = -1110.6 \times 10^{-6}$  in/in using the average value of the coefficient of thermal expansion using a third order polynomial. Considering the integral to be the more accurate calculation, the percentage difference would be

$$\begin{aligned}
|\epsilon_a| &= \left| \frac{-1105.9 \times 10^{-6} + 1110.6 \times 10^{-6}}{-1105.9 \times 10^{-6}} \right| \times 100 \\
&= 0.42775\%
\end{aligned}$$

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## INTERPOLATION

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Topic	Newton's Divided Difference Interpolation
Summary	Textbook notes on Newton's divided difference interpolation.
Major	Mechanical Engineering
Authors	Autar Kaw, Michael Keteltas
Last Revised	August 27, 2009
Web Site	<a href="http://numericalmethods.eng.usf.edu">http://numericalmethods.eng.usf.edu</a>

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