

## Chapter 06.00G

# Physical Problem for Regression Mechanical Engineering

### Problem Statement

To make the fulcrum (Figure 1) of a bascule bridge, a long hollow steel shaft called the trunnion is shrink fit into a steel hub. The resulting steel trunnion-hub assembly is then shrink fit into the girder of the bridge.



**Figure 1** Trunnion-Hub-Girder (THG) assembly.

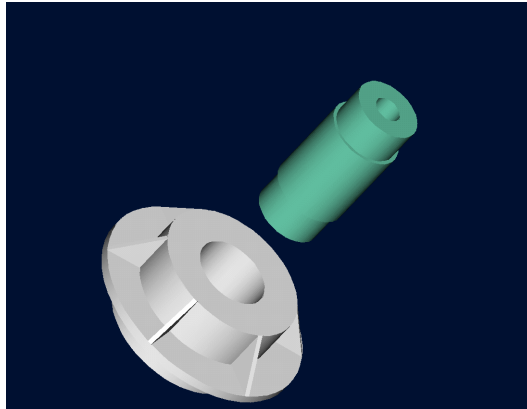
This is done by first immersing the trunnion in a cold medium such as dry-ice/alcohol mixture. After the trunnion reaches the steady state temperature of the cold medium, the trunnion outer diameter contracts. The trunnion is taken out of the medium and slid through the hole of the hub (Figure 2).

When the trunnion heats up, it expands and creates an interference fit with the hub. In 1995, on one of the bridges in Florida, this assembly procedure did not work as designed. Before the trunnion could be inserted fully into the hub, the trunnion got stuck. Luckily, the trunnion was taken out before it got stuck permanently. Otherwise, a new trunnion and hub would be needed to be ordered at a cost of \$50,000. Coupled with construction delays, the total loss could have been more than hundred thousand dollars.

Why did the trunnion get stuck? This was because the trunnion had not contracted enough to slide through the hole. Can you find out why?

A hollow trunnion of outside diameter 12.363" is to be fitted in a hub of inner diameter 12.358". The trunnion was put in dry ice/alcohol mixture (temperature of the fluid - dry ice/alcohol mixture is  $-108^{\circ}\text{F}$ ) to contract the trunnion so that it can be slid through

the hole of the hub. To slide the trunnion without sticking, a diametrical clearance of at least 0.01" is required between the trunnion and the hub. Assuming the room temperature is 80°F, is immersing it in dry-ice/alcohol mixture a correct decision?



**Figure 2** Trunnion slid through the hub after contracting.

### Solution

To calculate the contraction in the diameter of the trunnion, thermal expansion coefficient at room temperature is used. In that case the reduction,  $\Delta D$  in the outer diameter of the trunnion is

$$\Delta D = D\alpha\Delta T \quad (1)$$

where

$D$  = outer diameter of the trunnion,

$\alpha$  = coefficient of thermal expansion coefficient at room temperature, and

$\Delta T$  = change in temperature.

Given

$$D = 12.363''$$

$$\alpha = 6.817 \times 10^{-6} \text{ in/in/}^\circ\text{F at } 80^\circ\text{F}$$

$$\begin{aligned} \Delta T &= T_{fluid} - T_{room} \\ &= -108 - 80 \\ &= -188^\circ\text{F} \end{aligned}$$

where

$T_{fluid}$  = temperature of dry-ice/alcohol mixture,

$T_{room}$  = room temperature,

the reduction in the trunnion outer diameter is given by

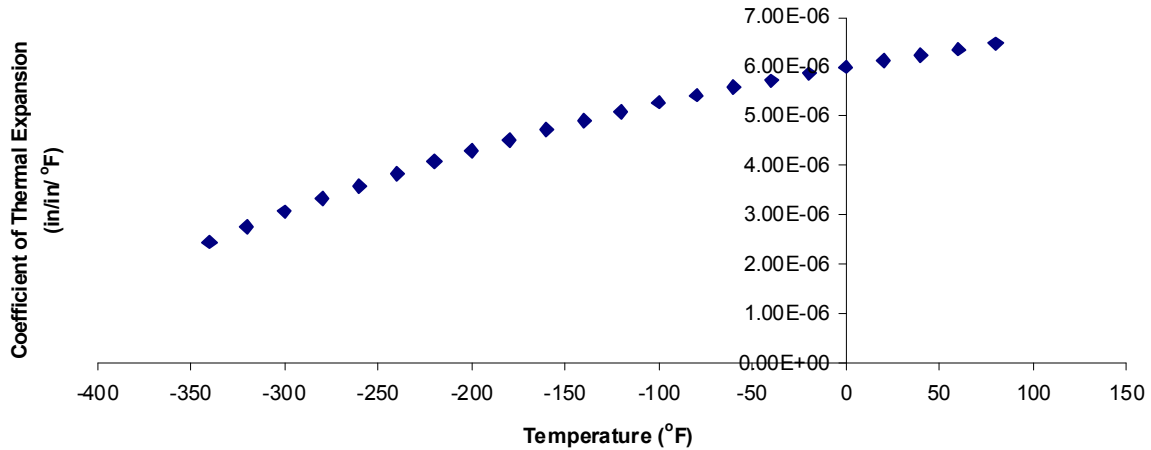
$$\begin{aligned} \Delta D &= (12.363)(6.47 \times 10^{-6})(-188) \\ &= -0.01504'' \end{aligned}$$

So the trunnion is predicted to reduce in diameter by 0.01504". But, is this enough reduction in diameter? As per specifications, he needs the trunnion to contract by

$$\begin{aligned} &= \text{trunnion outside diameter} - \text{hub inner diameter} + \text{diametric clearance} \\ &= 12.363'' - 12.358'' + 0.01'' \\ &= 0.015'' \end{aligned}$$

So according to his calculations, immersing the steel trunnion in dry-ice/alcohol mixture gives the desired contraction of 0.015" as we predict a contraction of 0.01504" .

But as shown in Figure 3 and Table 1, the thermal expansion coefficient of steel decreases with temperature and is not constant over the range of temperature the trunnion goes through. Hence, the above formula (Equation 1) would overestimate the thermal contraction.



**Figure 3** Varying thermal expansion coefficient as a function of temperature for cast steel.

The contraction in the diameter for the trunnion for which the thermal expansion coefficient varies as a function of temperature is given by

$$\Delta D = D \int_{T_{room}}^{T_{fluid}} \alpha dT \quad (2)$$

So one needs to find the curve to find the coefficient of thermal expansion as a function of temperature. This is done by regression where we best fit a curve through the data given in Table 1. In this case, we may fit a second order polynomial

$$\alpha = a_0 + a_1 \times T + a_2 \times T^2. \quad (3)$$

The values of the coefficients in the above equations will be found by polynomial regression. Knowing the values of  $a_0$ ,  $a_1$ , and  $a_2$ , we can then find the contraction in the trunnion diameter as

$$\begin{aligned} \Delta D &= D \int_{T_{room}}^{T_{fluid}} (a_0 + a_1 \times T + a_2 \times T^2) dT \\ &= D \left[ a_0 \times (T_{fluid} - T_{room}) + a_1 \times \frac{(T_{fluid}^2 - T_{room}^2)}{2} + a_2 \times \frac{(T_{fluid}^3 - T_{room}^3)}{3} \right] \end{aligned}$$

### Questions

1. Find the contraction in the outer diameter of the trunnion.
2. As required, is the magnitude of contraction more than 0.015" ?

3. If the magnitude of contraction is less than 0.015", what if the trunnion were immersed in liquid nitrogen (boiling temperature =  $-321^{\circ}\text{F}$ )? Will that give enough contraction in the trunnion?
4. Rather than regressing the data to a second order polynomial so that one can find the contraction in the outer diameter of the trunnion, how would you use Trapezoidal rule of integration for unequal segments to find the contraction? What is the relative difference between the two results? The data for the thermal expansion coefficient as a function of temperature is given in Table 1.
5. We chose a second order polynomial for regression. What would make a different order polynomial a better choice for regression? Is there an optimum order of polynomial you can find?

**Table 1** Instantaneous thermal expansion coefficient as a function of temperature.

Temperature	Instantaneous Thermal Expansion
$^{\circ}\text{F}$	$\mu\text{in}/\text{in}/^{\circ}\text{F}$
80	6.47
60	6.36
40	6.24
20	6.12
0	6.00
-20	5.86
-40	5.72
-60	5.58
-80	5.43
-100	5.28
-120	5.09
-140	4.91
-160	4.72
-180	4.52
-200	4.30
-220	4.08
-240	3.83
-260	3.58
-280	3.33
-300	3.07
-320	2.76
-340	2.45

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**REGRESSION**

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Topic	A physical problem for regression.
Summary	Finding the regression model for thermal expansion coefficient as a function of temperature to calculate the contraction in the diameter of a trunnion for purposes of shrink fitting.
Major	Mechanical Engineering
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