Topic : Lagrangian Method - Interpolation
Simulation : Graphical Simulation of the Method
Language : Matlab r12
Authors : Nathan Collier, Autar Kaw
Date : 1 November 2002
Abstract : This simulation illustrates the Lagrangian method of interpolation. Given n data points of y versus x, you are then required to find the value of y at a particular value of x using first, second, and third order interpolation. So one has to first pick the needed data points, and then use those to interpolate data.

```
clear all
% INPUTS: Enter the following

% Array of x-data
  x=[10 0 20 15 30 22.5];

% Array of y-data
  y=[227.04 0 517.35 362.78 901.67 602.97];

% Value of x at which y is desired
  xdesired = 16;

% SOLUTION

% This calculates window size to be used in figures
set(0,'Units','pixels')
scnsize = get(0,'ScreenSize');
wid = round(scnsize(3));
hei = round(0.95*scnsize(4));
wind = [1, 1, wid, hei];

% The following considers the x and y data and selects the two closest points to xdesired
% that also bracket it.
n = numel(x);
comp = abs(x-xdesired);
c=min(comp);
for i=1:n
    if comp(i)==c;
    ci=i;
    end
end
if x(ci) < xdesired
    q=1;
    for i=1:n
        if x(i) > xdesired
            ne(q)=x(i);
```

```
            q=q+1;
        end
    end
    b=min(ne);
    for i=1:n
        if x(i)==b
            bi=i;
        end
    end
end
if x(ci) > xdesired
    q=1;
    for i=1:n
        if x(i) < xdesired
            ne(q)=x(i);
            q=q+1;
        end
    end
    b=max(ne);
    for i=1:n
        if x(i)==b
            bi=i;
        end
    end
end
% If more than two values are needed, the following selects the subsequent values and
puts
% them into a matrix, preserving the orginal data order.
for i = 1:n
    A(i,2)=i;
    A(i,1)=comp(i);
end
A=sortrows(A,1);
for i=1:n
    A(i,3)=i;
end
A=sortrows(A,2);
d=A(1:n,3);
if d(bi)~=2
    temp=d(bi);
    d(bi)=1;
    for i=1:n
        if i ~= bi & i ~= ci & d(i) <= temp
            d(i)=d(i)+1;
        end
        d(ci)=1;
```

```
   end
end

%%%%%%%%% LINEAR INTERPOLATION %%%%%%%%%

% Pick two data points
datapoints=2;
p=1;
for i=1:n
   if d(i) <= datapoints
      xdata(p)=x(i);
      ydata(p)=y(i);
      p=p+1;
   end
end

% Setting up the Lagrangian polynomial
z=sym('z');
L0=(z-xdata(2))/(xdata(1)-xdata(2));
L1=(z-xdata(1))/(xdata(2)-xdata(1));
fl=L0*ydata(1)+L1*ydata(2);
fxdesired=subs(fl,z,xdesired);
fprev=fxdesired;


% Writing to figure windows
figure('Position',wind)
s=0.04;
title('Linear interpolation','Fontweight','bold','FontSize',14)
text(0,1,'Selected Data','Fontweight','bold')
axis off
p=1;
text(0,p-s ,['      ',num2str(xdata(1))])
text(0,p-1.5*s,['x = '])
text(0,p-2*s,['      ',num2str(xdata(2))])
text(0.1,p-s ,['      ',num2str(ydata(1))])
text(0.1,p-1.5*s,['y = '])
text(0.1,p-2*s,['      ',num2str(ydata(2))])

p=p-2*s-2*s;
text(0,p,'Setting up the Lagrangian polynomial','Fontweight','bold')
text(0,p-s,'L0(x) = (x-x(2))/(x(1)-x(2))')
text(0,p-2*s,'L1(x) = (x-x(1))/(x(2)-x(1))')
text(0,p-4*s,'f(x) = L0(x)*y(1) + L1(x)*y(2)')

p=p-6*s;
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```
text(0,p,'Calculating value at desired point','Fontweight','bold')
text(0,p-s,['f(xdesired) = f(',num2str(xdesired),') = ',num2str(fxdesired)])

figure('Position',wind)
axis on
subplot(2,1,1), ezplot(fl,[min(xdata),max(xdata)])
title('Linear interpolation','Fontweight','bold')
hold on
plot(xdata,ydata,'ro','MarkerSize',10','MarkerFaceColor',[1,0,0])
plot(xdesired,fxdesired,'kx','Linewidth',2,'MarkerSize',12')

axis on
subplot(2,1,2), ezplot(fl,[min(xdata),max(xdata)])
title('  ')
hold on
plot(x,y,'ro','MarkerSize',10','MarkerFaceColor',[1,0,0])
plot(xdesired,fxdesired,'kx','Linewidth',2,'MarkerSize',12')
xlim([min(x) max(x)])
ylim([min(y) max(y)])


%%%%%%%% QUADRATIC INTERPOLATION %%%%%%%%%

% Pick three data points
datapoints=3;
p=1;
for i=1:n
   if d(i) <= datapoints
      xdata(p)=x(i);
      ydata(p)=y(i);
      p=p+1;
   end
end

% Calculating coefficients of Newton's Divided difference polynomial
% Setting up the Lagrangian polynomial
z=sym('z');
L0=((z-xdata(2))*(z-xdata(3)))/((xdata(1)-xdata(2))*(xdata(1)-xdata(3)));
L1=((z-xdata(1))*(z-xdata(3)))/((xdata(2)-xdata(1))*(xdata(2)-xdata(3)));
L2=((z-xdata(1))*(z-xdata(2)))/((xdata(3)-xdata(1))*(xdata(3)-xdata(2)));
fq=L0*ydata(1)+L1*ydata(2)+L2*ydata(3);
fxdesired=subs(fq,z,xdesired);

fnew=fxdesired;
ea=abs((fnew-fprev)/fnew*100);
if ea >= 5
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   sd=0;
else
   sd=floor(2-log10(abs(ea)/0.5));
end

% Writing to figure windows
figure('Position',wind)
s=0.04;
title('Quadratic interpolation','Fontweight','bold','FontSize',14)
text(0,1,'Selected Data','Fontweight','bold')
axis off
p=1;
text(0,p-s ,['      ',num2str(xdata(1))])
text(0,p-1.5*s,['x = '])
text(0,p-2*s,['      ',num2str(xdata(2))])
text(0,p-3*s,['      ',num2str(xdata(3))])
text(0.1,p-s ,['      ',num2str(ydata(1))])
text(0.1,p-1.5*s,['y = '])
text(0.1,p-2*s,['      ',num2str(ydata(2))])
text(0.1,p-3*s,['      ',num2str(ydata(3))])

p=p-3*s-2*s;
text(0,p,'Setting up the Lagrangian polynomial','Fontweight','bold')
text(0,p-s,'L0(x) = ((x-x(2))*(x-x(3)))/((x(1)-x(2))*(x(1)-x(3)))')
text(0,p-2*s,['        = ((x-',num2str(xdata(2)),')*(x-
',num2str(xdata(3)),'))/((',num2str(xdata(1)),'-',num2str(xdata(2)),')*(',num2str(xdata(1)),'-
',num2str(xdata(3)),'))'])
text(0,p-3*s,'L1(x) = ((x-x(1))*(x-x(3)))/((x(2)-x(1))*(x(2)-x(3)))')
text(0,p-4*s,['        = ((x-',num2str(xdata(1)),')*(x-
',num2str(xdata(3)),'))/((',num2str(xdata(2)),'-',num2str(xdata(1)),')*(',num2str(xdata(2)),'-
',num2str(xdata(3)),'))'])
text(0,p-5*s,'L2(x) = ((x-x(1))*(x-x(2)))/((x(3)-x(1))*(x(3)-x(2)))')
text(0,p-6*s,['        = ((x-',num2str(xdata(1)),')*(x-
',num2str(xdata(2)),'))/((',num2str(xdata(3)),'-',num2str(xdata(1)),')*(',num2str(xdata(3)),'-
',num2str(xdata(2)),'))'])
text(0,p-8*s,'f(x) = L0(x)*y(1) + L1(x)*y(2) + L2(x)*y(3)')

p=p-10*s;
text(0,p,'Calculating value at desired point','Fontweight','bold')
text(0,p-s,['f(xdesired) = f(',num2str(xdesired),') = ',num2str(fxdesired)])

p=p-3*s;
text(0,p,'Absolute relative approximate error and significant digits','Fontweight','bold')
text(0,p-s,['ea = abs(( ',num2str(fnew),' - ',num2str(fprev),' )/ ',num2str(fnew),')*100 =
',num2str(ea), ' %'])
text(0,p-2*s,['sigdig = ',num2str(sd)])
```

```
figure('Position',wind)
axis on
subplot(2,1,1), ezplot(fq,[min(xdata),max(xdata)])
title('Quadratic interpolation','Fontweight','bold')
hold on
plot(xdata,ydata,'ro','MarkerSize',10','MarkerFaceColor',[1,0,0])
plot(xdesired,fxdesired,'kx','Linewidth',2,'MarkerSize',12')

axis on
subplot(2,1,2), ezplot(fq,[min(xdata),max(xdata)])
title('  ')
hold on
plot(x,y,'ro','MarkerSize',10','MarkerFaceColor',[1,0,0])
plot(xdesired,fxdesired,'kx','Linewidth',2,'MarkerSize',12')
xlim([min(x) max(x)])
ylim([min(y) max(y)])
fprev=fnew;


%%%%%%%%% CUBIC INTERPOLATION %%%%%%%%%

% Pick four data points
datapoints=4;
p=1;
for i=1:n
   if d(i) <= datapoints
      xdata(p)=x(i);
      ydata(p)=y(i);
      p=p+1;
   end
end

% Calculating coefficients of Newton's Divided difference polynomial
z=sym('z');
L0=((z-xdata(2))*(z-xdata(3))*(z-xdata(4)))/((xdata(1)-xdata(2))*(xdata(1)-
xdata(3))*(xdata(1)-xdata(4)));
L1=((z-xdata(1))*(z-xdata(3))*(z-xdata(4)))/((xdata(2)-xdata(1))*(xdata(2)-
xdata(3))*(xdata(2)-xdata(4)));
L2=((z-xdata(1))*(z-xdata(2))*(z-xdata(4)))/((xdata(3)-xdata(1))*(xdata(3)-
xdata(2))*(xdata(3)-xdata(4)));
L3=((z-xdata(1))*(z-xdata(2))*(z-xdata(3)))/((xdata(4)-xdata(1))*(xdata(4)-
xdata(2))*(xdata(4)-xdata(3)));

fc=L0*ydata(1)+L1*ydata(2)+L2*ydata(3)+L3*ydata(4);
fxdesired=subs(fc,z,xdesired);
```

```
fnew=fxdesired;
ea=abs((fnew-fprev)/fnew*100);
if ea >= 5
   sd=0;
else
   sd=floor(2-log10(abs(ea)/0.5));
end

figure('Position',wind)
title('Cubic interpolation','Fontweight','bold','FontSize',14)
text(0,1,'Selected Data','Fontweight','bold')
axis off
p=1;
text(0,p-s ,['      ',num2str(xdata(1))])
text(0,p-1.5*s,['x = '])
text(0,p-2*s,['      ',num2str(xdata(2))])
text(0,p-3*s,['      ',num2str(xdata(3))])
text(0,p-4*s,['      ',num2str(xdata(4))])
text(0.1,p-s ,['      ',num2str(ydata(1))])
text(0.1,p-1.5*s,['y = '])
text(0.1,p-2*s,['      ',num2str(ydata(2))])
text(0.1,p-3*s,['      ',num2str(ydata(3))])
text(0.1,p-4*s,['      ',num2str(ydata(4))])

p=p-4*s-2*s;
text(0,p,'Setting up the Lagrangian polynomial','Fontweight','bold')
text(0,p-s,'L0(x) = ((x-x(2))*(x-x(3))*(x-x(4)))/((x(1)-x(2))*(x(1)-x(3))*(x(1)-x(4)))')
text(0,p-2*s,'L1(x) = ((x-x(1))*(x-x(3))*(x-x(4)))/((x(2)-x(1))*(x(2)-x(3))*(x(2)-x(4)))')
text(0,p-3*s,'L2(x) = ((x-x(1))*(x-x(2))*(x-x(4)))/((x(3)-x(1))*(x(3)-x(2))*(x(3)-x(4)))')
text(0,p-4*s,'L3(x) = ((x-x(1))*(x-x(2))*(x-x(3)))/((x(4)-x(1))*(x(4)-x(2))*(x(4)-x(3)))')
text(0,p-6*s,'f(x) = L0*y(1)+L1*y(2)+L2*y(3)+L3*y(4)')

p=p-8*s;
text(0,p,'Calculating value at desired point','Fontweight','bold')
text(0,p-s,['f(xdesired) = f(',num2str(xdesired),') = ',num2str(fxdesired)])

p=p-3*s;
text(0,p,'Absolute relative approximate error and significant digits','Fontweight','bold')
text(0,p-s,['ea = abs(( ',num2str(fnew),' - ',num2str(fprev),' )/ ',num2str(fnew),')*100 =
',num2str(ea), ' %'])
text(0,p-2*s,['sigdig = ',num2str(sd)])

figure('Position',wind)
axis on
subplot(2,1,1), ezplot(fc,[min(xdata),max(xdata)])
```

```
title('Cubic interpolation','Fontweight','bold')
hold on
plot(xdata,ydata,'ro','MarkerSize',10','MarkerFaceColor',[1,0,0])
plot(xdesired,fxdesired,'kx','Linewidth',2,'MarkerSize',12')

axis on
subplot(2,1,2), ezplot(fc,[min(xdata),max(xdata)])
title('  ')
hold on
plot(x,y,'ro','MarkerSize',10','MarkerFaceColor',[1,0,0])
plot(xdesired,fxdesired,'kx','Linewidth',2,'MarkerSize',12')
xlim([min(x) max(x)])
ylim([min(y) max(y)])
```

Figure No. 1

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Linear interpolation

Selected Data

$x = \begin{matrix} 20 \\ 15 \end{matrix}$ $y = \begin{matrix} 517.35 \\ 362.78 \end{matrix}$

Setting up the Lagrangian polynomial

L0(x) = (x-x(2))/(x(1)-x(2))

L1(x) = (x-x(1))/(x(2)-x(1))

f(x) = L0(x)*y(1) + L1(x)*y(2)

Calculating value at desired point

f(xdesired) = f(16) = 393.694

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Figure No. 2

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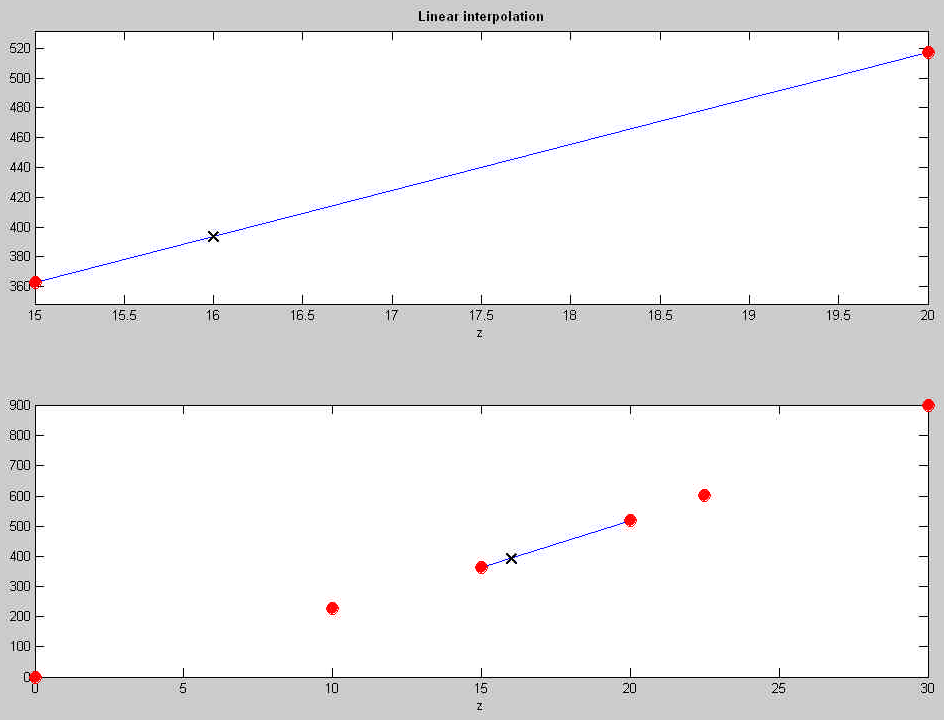

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Figure No. 3

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Quadratic interpolation

Selected Data

$$x = \begin{matrix} 10 \\ 20 \\ 15 \end{matrix} \qquad y = \begin{matrix} 227.04 \\ 517.35 \\ 362.78 \end{matrix}$$

Setting up the Lagrangian polynomial

L0(x) = ((x-x(2))*(x-x(3)))/((x(1)-x(2))*(x(1)-x(3)))

= ((x-20)*(x-15))/((10-20)*(10-15))

L1(x) = ((x-x(1))*(x-x(3)))/((x(2)-x(1))*(x(2)-x(3)))

= ((x-10)*(x-15))/((20-10)*(20-15))

L2(x) = ((x-x(1))*(x-x(2)))/((x(3)-x(1))*(x(3)-x(2)))

= ((x-10)*(x-20))/((15-10)*(15-20))

f(x) = L0(x)*y(1) + L1(x)*y(2) + L2(x)*y(3)

Calculating value at desired point

f(xdesired) = f(16) = 392.1876

Absolute relative approximate error and significant digits

ea = abs((392.1876 - 393.694)/ 392.1876)*100 = 0.3841 %

sigdig = 2

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Figure No. 4

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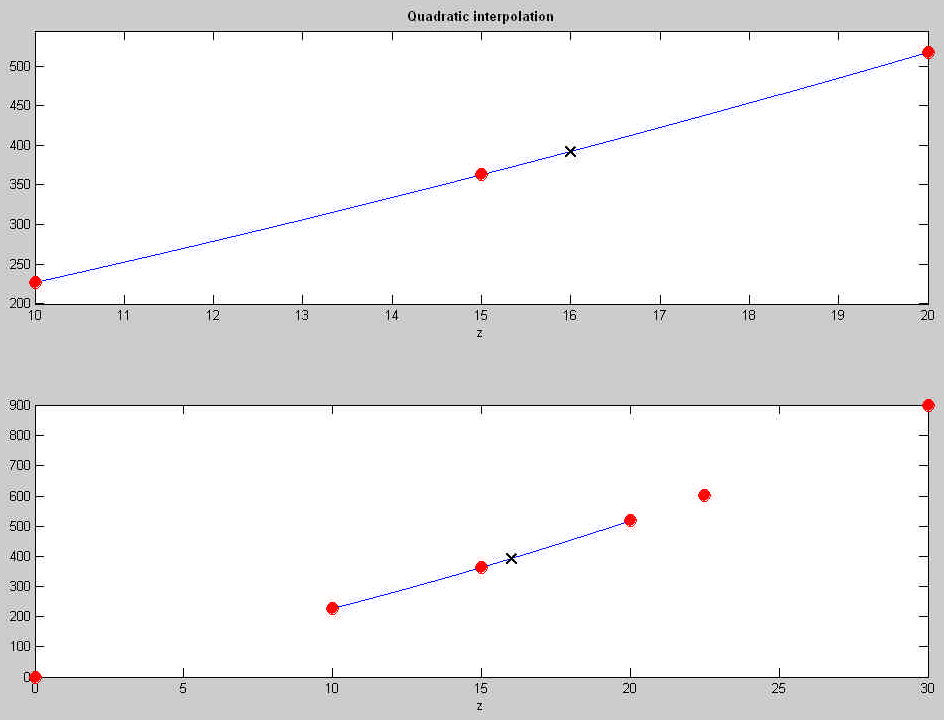

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Figure No. 5

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Cubic interpolation

Selected Data

x =	10	y = 227.04
	20	517.35
	15	362.78
	22.5	602.97

Setting up the Lagrangian polynomial

L0(x) = ((x-x(2))*(x-x(3))*(x-x(4)))/((x(1)-x(2))*(x(1)-x(3))*(x(1)-x(4)))

L1(x) = ((x-x(1))*(x-x(3))*(x-x(4)))/((x(2)-x(1))*(x(2)-x(3))*(x(2)-x(4)))

L2(x) = ((x-x(1))*(x-x(2))*(x-x(4)))/((x(3)-x(1))*(x(3)-x(2))*(x(3)-x(4)))

L3(x) = ((x-x(1))*(x-x(2))*(x-x(3)))/((x(4)-x(1))*(x(4)-x(2))*(x(4)-x(3)))

f(x) = L0*y(1)+L1*y(2)+L2*y(3)+L3*y(4)

Calculating value at desired point

f(xdesired) = f(16) = 392.0572

Absolute relative approximate error and significant digits

ea = abs((392.0572 - 392.1876)/ 392.0572)*100 = 0.033269 %

sigdig = 3

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Figure No. 6

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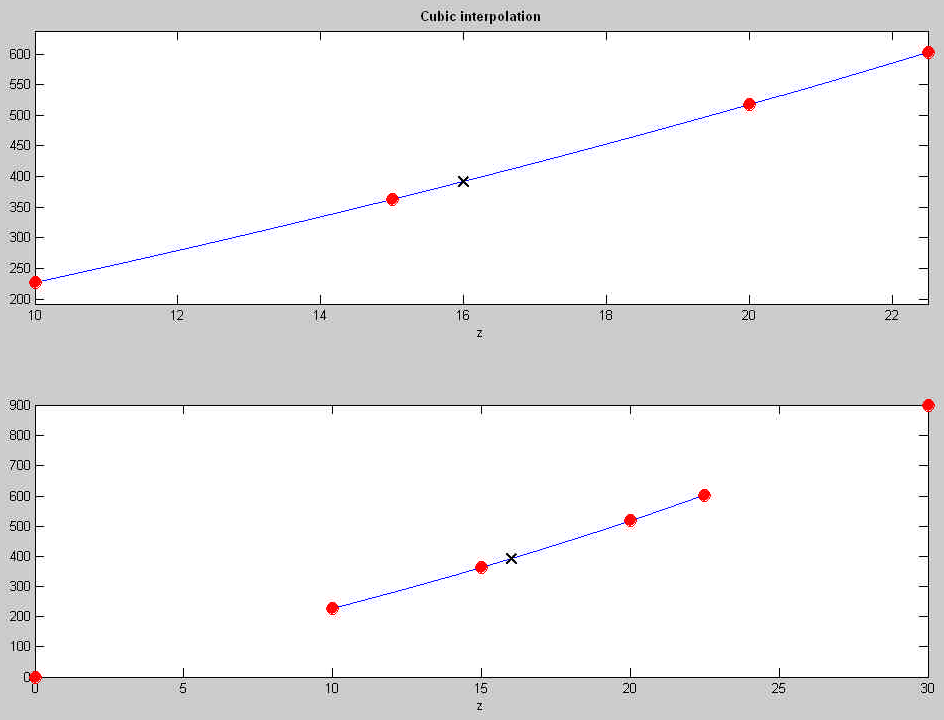

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