

Topic : Additional Interpolation Topics

Simulation : Higher Order Interpolation is a Bad Idea

Language : Matlab r12

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Abstract : In 1901, Carl Runge published his work on dangers of higher order polynomial interpolation. He took a simple looking function $f(x)=1/(1+25x^2)$ on the interval of $[-1,1]$. He took points equidistantly spaced in $[-1,1]$ and interpolated the points with polynomials. He found that as he took more points, the polynomials and the original curve differed considerably in their value.

```
clear all
```

```
% INPUTS: Enter the following
```

```
% You can make three separate choices for the number of equidistant points in [-1,1].
```

```
So
```

```
% if you choose n1=5, you are using a 4th order polynomial to approximate
```

```
1/(1+25x^2) in
```

```
% [-1,1].
```

```
n1=5;
```

```
n2=9;
```

```
n3=17;
```

```
% SOLUTION:
```

```
% This calculates window size to be used in figures
```

```
set(0,'Units','pixels')
```

```
scnsize = get(0,'ScreenSize');
```

```
wid = round(scnsize(3));
```

```
hei = round(0.95*scnsize(4));
```

```
wind = [1, 1, wid, hei];
```

```
% Plotting Runge's Function
```

```
figure('Position',wind)
```

```
frunge=inline('1/(1+25*x^2)');
```

```
n=1;
```

```
for i =-1:0.01:1
```

```
    x(n)=i;
```

```
    y(n)=frunge(x(n));
```

```
    n=n+1;
```

```
end
```

```
plot(x,y,'k','LineWidth',3)
```

```
title('Runge^s function and three interpolants using equidistantly spaced  
points','Fontweight','bold','FontSize',14)
```

```
hold on
```

```
% Performing each of the three polynomial fits
```

```

% n1 polynomial fit
for i=1:n1
    x1(i)=2/(n1-1)*(i-1)-1;
    y1(i)=frunge(x1(i));
end
p1=polyfit(x1,y1,n1-1);
x1p=-1:0.01:1;
y1p=polyval(p1,x1p);
plot(x1p,y1p,'b','LineWidth',3)

% n2 polynomial fit
for i=1:n2
    x2(i)=2/(n2-1)*(i-1)-1;
    y2(i)=frunge(x2(i));
end
p2=polyfit(x2,y2,n2-1);
x2p=-1:0.01:1;
y2p=polyval(p2,x2p);
plot(x2p,y2p,'r','LineWidth',3)

% n3 polynomial fit
for i=1:n3
    x3(i)=2/(n3-1)*(i-1)-1;
    y3(i)=frunge(x3(i));
end
p3=polyfit(x3,y3,n3-1);
x3p=-1:0.01:1;
y3p=polyval(p3,x3p);
plot(x3p,y3p,'g','LineWidth',3)

legend('Runge^s Function','n1','n2','n3',4)
ylim([-1.5,1.5])
xlim([-1,1])

% Constructing and printing the three tables of values
% n1's table
for i = 1 : 2*(n1-1)+1
    xi(i)=2/(2*(n1-1))*(i-1)-1;
    M(i,1)=xi(i);
    M(i,2)=frunge(xi(i));
    M(i,3)=polyval(p1,xi(i));
    M(i,4)=abs((frunge(xi(i))-polyval(p1,xi(i)))/frunge(xi(i)))*100;
    if M(i,4)<1*10^-7
        M(i,4)=0;
    end
end
end

```

```

figure('Position',wind)
for i=1:2*n1
    for j=1:4
        rectangle('Position',[(j-1)*0.25,1-(i-1)/35,0.25,1/35])
        if i == 1 & j == 1
            text((j-1)*0.25+0.02,1-(i-1)/35+1/70,'X')
            text((j)*0.25+0.02,1-(i-1)/35+1/70,'Actual')
            text((j+1)*0.25+0.02,1-(i-1)/35+1/70,'Interpolated')
            text((j+2)*0.25+0.02,1-(i-1)/35+1/70,'% True Error')
        end
        if i >= 2
            text((j-1)*0.25+0.02,1-(i-1)/35+1/70,[num2str(M(i-1,j))])
        end
    end
end
title(['Table of Values for n1 = ',num2str(n1)],'Fontweight','bold','FontSize',14)
axis off
ylim([0,1])
xlim([0,1])

```

```

% n2's table
for i = 1 : 2*(n2-1)+1
    xi(i)=2/(2*(n2-1))*(i-1)-1;
    M(i,1)=xi(i);
    M(i,2)=frunge(xi(i));
    M(i,3)=polyval(p2,xi(i));
    M(i,4)=abs((frunge(xi(i))-polyval(p2,xi(i)))/frunge(xi(i)))*100;
    if M(i,4)<1*10^-7
        M(i,4)=0;
    end
end
end

```

```

figure('Position',wind)
for i=1:2*n2
    for j=1:4
        rectangle('Position',[(j-1)*0.25,1-(i-1)/35,0.25,1/35])
        if i == 1 & j == 1
            text((j-1)*0.25+0.02,1-(i-1)/35+1/70,'X')
            text((j)*0.25+0.02,1-(i-1)/35+1/70,'Actual')
            text((j+1)*0.25+0.02,1-(i-1)/35+1/70,'Interpolated')
            text((j+2)*0.25+0.02,1-(i-1)/35+1/70,'% True Error')
        end
        if i >= 2
            text((j-1)*0.25+0.02,1-(i-1)/35+1/70,[num2str(M(i-1,j))])
        end
    end
end

```

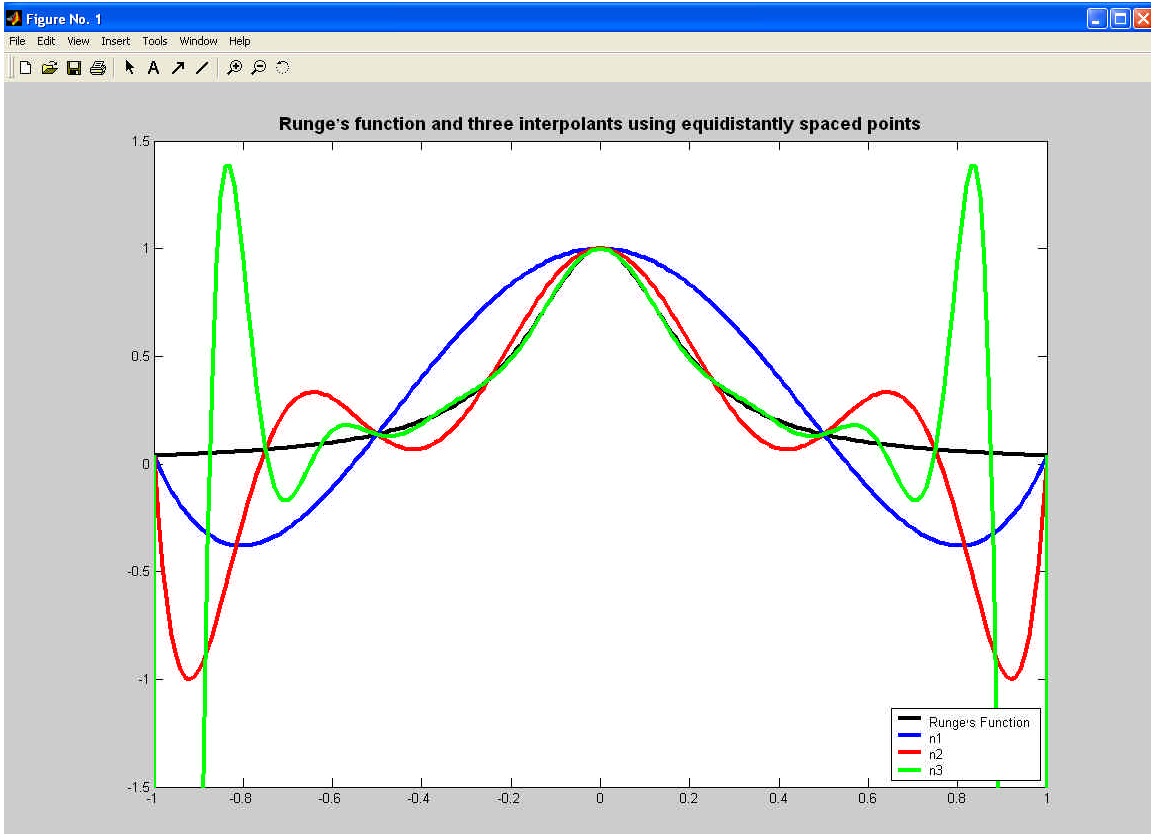
```

    end
end
title(['Table of Values for n2 = ',num2str(n2)],'Fontweight','bold','FontSize',14)
axis off
ylim([0,1])
xlim([0,1])

% n3's table
for i = 1 : 2*(n3-1)+1
    xi(i)=2/(2*(n3-1))*(i-1)-1;
    M(i,1)=xi(i);
    M(i,2)=frunge(xi(i));
    M(i,3)=polyval(p3,xi(i));
    M(i,4)=abs((frunge(xi(i))-polyval(p3,xi(i)))/frunge(xi(i)))*100;
    if M(i,4)<1*10^-7
        M(i,4)=0;
    end
end
end

figure('Position',wind)
for i=1:2*n3
    for j=1:4
        rectangle('Position',[(j-1)*0.25,1-(i-1)/35,0.25,1/35])
        if i == 1 & j == 1
            text((j-1)*0.25+0.02,1-(i-1)/35+1/70,'X')
            text((j)*0.25+0.02,1-(i-1)/35+1/70,'Actual')
            text((j+1)*0.25+0.02,1-(i-1)/35+1/70,'Interpolated')
            text((j+2)*0.25+0.02,1-(i-1)/35+1/70,'% True Error')
        end
        if i >= 2
            text((j-1)*0.25+0.02,1-(i-1)/35+1/70,[num2str(M(i-1,j))])
        end
    end
end
end
title(['Table of Values for n3 = ',num2str(n3)],'Fontweight','bold','FontSize',14)
axis off
ylim([0,1])
xlim([0,1])

```



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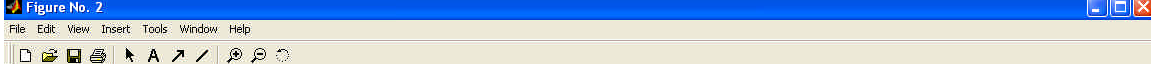


Table of Values for $n_1 = 5$

X	Actual	Interpolated	% True Error
-1	0.038462	0.038462	0
-0.75	0.06639	-0.35683	637.4693
-0.5	0.13793	0.13793	0
-0.25	0.39024	0.74563	91.067
0	1	1	0
0.25	0.39024	0.74563	91.067
0.5	0.13793	0.13793	0
0.75	0.06639	-0.35683	637.4693
1	0.038462	0.038462	0

start MATLAB C:\Documents a... Figure No. 1 Figure No. 2 Figure No. 3 Figure No. 4 Document1 - Mic... 1:23 AM

Table of Values for $n2 = 9$

X	Actual	Interpolated	% True Error
-1	0.038462	0.038462	0
-0.875	0.049651	-0.63102	1773.7199
-0.75	0.06639	0.06639	0
-0.625	0.092888	0.32826	253.3886
-0.5	0.13793	0.13793	0
-0.375	0.22145	0.09196	58.4743
-0.25	0.39024	0.39024	0
-0.125	0.7191	0.6083	12.4036
0	1	1	0
0.125	0.7191	0.6083	12.4036
0.25	0.39024	0.39024	0
0.375	0.22145	0.09196	58.4743
0.5	0.13793	0.13793	0
0.625	0.092888	0.32826	253.3886
0.75	0.06639	0.06639	0
0.875	0.049651	-0.63102	1773.7199
1	0.038462	0.038462	0

Table of Values for $n3 = 17$

X	Actual	Interpolated	% True Error
-1	0.038462	0.038462	0
-0.9375	0.04353	-10.1739	23472.0754
-0.875	0.049651	0.049651	0
-0.8125	0.05713	1.1818	1968.6257
-0.75	0.06639	0.06639	0
-0.6875	0.078025	-0.14606	287.2
-0.625	0.092888	0.092888	0
-0.5625	0.11223	0.1806	60.9212
-0.5	0.13793	0.13793	0
-0.4375	0.17286	0.14337	17.0579
-0.375	0.22145	0.22145	0
-0.3125	0.29058	0.30751	5.8272
-0.25	0.39024	0.39024	0
-0.1875	0.53222	0.52071	2.1644
-0.125	0.7191	0.7191	0
-0.0625	0.91103	0.91622	0.56958
0	1	1	0
0.0625	0.91103	0.91622	0.56958
0.125	0.7191	0.7191	0
0.1875	0.53222	0.52071	2.1644
0.25	0.39024	0.39024	0
0.3125	0.29058	0.30751	5.8272
0.375	0.22145	0.22145	0
0.4375	0.17286	0.14337	17.0579
0.5	0.13793	0.13793	0
0.5625	0.11223	0.1806	60.9212
0.625	0.092888	0.092888	0
0.6875	0.078025	-0.14606	287.2
0.75	0.06639	0.06639	0
0.8125	0.05713	1.1818	1968.6257
0.875	0.049651	0.049651	0
0.9375	0.04353	-10.1739	23472.0754
1	0.038462	0.038462	0