Topic : Additional Interpolation Topics
Simulation : The Effect of Choice of Points on Interpolation
Language : Matlab r12
Authors : Nathan Collier, Autar Kaw
Date : 12 November 2002
Abstract : In 1901, Carl Runge published his work on dangers of higher order interpolation. He took a simple looking function $\mathrm{f}(\mathrm{x})=1 /\left(1+25 \mathrm{x}^{2}\right)$ on the interval $[-1,1]$. He took points equidistantly spaced in $[-1,1]$ and interpolated the points with polynomials. He found that as he took more points, the polynomials and the original curve differed considerably. However, if he took data points close to the ends of the interval [-1,1], the problem of large differences between interpolated and actual values was less pronounced. This simulation shows you this phenomena.
clear all
\% INPUTS: Enter the following
\% Enter the number of points chosen for interpolation in [-1,1]
$\mathrm{n}=13$;
\% NOTE: For the number scheme to work properly n needs to be an odd number

## \% SOLUTION:

\% This calculates window size to be used in figures set(0,'Units','pixels')
scnsize = get( 0 ,'ScreenSize');
wid $=$ round (scnsize(3));
hei $=\operatorname{round}(0.95 * \operatorname{scnsize}(4))$;
wind $=[1,1$, wid, hei $]$;
figure('Position', wind)
frunge=inline(' $1 /\left(1+25 *^{\wedge} \wedge 2\right)$ ');
ezplot(frunge)
title('Runge^,s function and interpolant with equidistantly spaced
points','Fontweight','bold','FontSize',14)
hold on
$x \lim ([-1,1])$
$y \lim ([-1.1,1.1])$
for $\mathrm{i}=1$ : n
$X(\mathrm{i})=2 /(\mathrm{n}-1) *(\mathrm{i}-1)-1$;
$\mathrm{Y}(\mathrm{i})=$ frunge $(\mathrm{X}(\mathrm{i})$ );
end
$\operatorname{plot}(X, Y$, 'ro','MarkerSize',, ,'MarkerFaceColor',[1,0,0])
$\mathrm{p}=$ polyfit(X,Y,numel(X)-1);
$\mathrm{x} 1=-1: 0.01: 1$;
$\mathrm{y} 1=\operatorname{polyval}(\mathrm{p}, \mathrm{x} 1)$;

```
plot(x1,y1,'k')
figure('Position',wind)
ezplot(frunge)
title('Runge^,s function and interpolant with more points near the end of the interval [-
1,1]','Fontweight','bold','FontSize',14)
hold on
xlim([-1,1])
ylim([-1.1,1.1])
d=0;
for i=2:(n-1)/2+1
    d=d+2^(i-2);
end
l=1/d;
Xb}(1)=-1
for i=2:(n-1)/2+1
    Xb(i)=Xb(i-1)+2^(i-2)*I;
end
for i = (n-1)/2+2:n
    Xb(i)=Xb(i-1)+2^(n-i)*l;
end
for i=1:n
    Yb(i)=frunge(Xb(i));
end
plot(Xb,Yb,'ro','MarkerSize',8,'MarkerFaceColor',[1,0,0])
p=polyfit(Xb,Yb,numel(X)-1);
x1=-1:0.01:1;
yl=polyval(p,x1);
plot(x1,y1,'k')
```



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Runge's function and interpolant with more points near the end of the interval [-1,1]


